UNIVERSITY OF KELANIYA-SRI LANKA

Centre for Distance & Continuing Education

Bachelor of Science (General) External

Third Year Second Semester Examination -2025 February

Pure Mathematics

PMAT 37632- Geometry

No.of Questions: Five (05) **No.of Pages : Three(03)** **Time: Two(02)hrs**

Answer <u>Four(04)</u> Questions Only.

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01. (a) Consider the general second-degree equation

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

where a, b, c, f, g and h are constants.

Show that the condition for the above equation to represent a pair of straight lines is

$$\begin{vmatrix} a & h & f \\ h & b & g \\ f & g & c \end{vmatrix} = 0.$$

Let the equation $\lambda x^2 - 10xy + 12y^2 + 5x - 16y - 3 = 0$ represents a pair of straight lines. Find

(i) the value of λ and the separate equations of the lines

(ii) point of intersection of the lines.

(b) Show that the lines joining origin to the points of intersection of the curve

$S \equiv 3x^2 + 5xy - 3y^2 + 2x + 3y = 0$ and the line $3x - 2y - 1 = 0$ are perpendicular.

(c) Show that the product of perpendicular distances from the point (x_1, y_1) to the lines represented by $ax^2 + 2hxy + by^2 = 0$ is given by

$$\frac{|ax_1^2 + 2hx_1y_1 + by_1^2|}{\sqrt{(a-b)^2 + 4h^2}}.$$

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02. (a) The conic represented by $y^2 + 2y - 8x + 25 = 0$ can be transformed in to the form $y^2 = 4ax$ without rotating the axes.

(i) Find the coordinates of the new origin of the translated conic.

(ii) Determine the constant a.

(b) Using suitable transformations, show that

$S \equiv 14x^2 - 4xy + 11y^2 - 36x + 48y + 41 = 0$ represents an ellipse.

Find the centre, lengths of major axis and minor axis and the equations of major and minor axes.

(c) Show that the equation $S \equiv 16x^2 - 24xy + 9y^2 - 104x - 172y + 44 = 0$ represents a parabola using a suitable transformation. Find the equations of axes and the directrix of the parabola.

03. (a) Find the equations of tangent and normal to $2x^2 + 5xy + 3y^2 + 4x - 10y - 4 = 0$ at $(1,1)$.

(b) Show that $3x + 4y + 7 = 0$ touches the circle $x^2 + y^2 - 4x - 6y - 12 = 0$. Find the point of contact.

(c) If the straight line $\lambda x + \mu y + \nu = 0$ touches the parabola $y^2 - 4px + 4pq = 0$, then prove that $\lambda^2 q + \lambda \nu - p\mu^2 = 0$.

(d) The polar of the point P with respect to the circle $x^2 + y^2 = a^2$ touches the circle $4x^2 + 4y^2 = a^2$. Show that the locus of P is the circle $x^2 + y^2 = 4a^2$.

04. (a) Find the symmetrical form of the equation of the line $3x - 2y - z - 4 = 0$ and $4x + y - 2z - 3 = 0$.

(b) Show that the lines $\frac{x-1}{2} = \frac{y-3}{4} = \frac{z}{-1}$ and $\frac{x-4}{3} = \frac{y-2}{-2} = \frac{z-1}{1}$ are intersecting. Find the point of intersection.

(c) Find the distance of a point $(0,2,3)$ to the line $\frac{x+3}{3} = \frac{y-1}{2} = \frac{z+4}{3}$.

(d) Find the value of k so that the two lines $\frac{x-1}{3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y+1}{-3} = \frac{z+10}{8}$.

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05. (a) Find the equation of the plane containing the line $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$ and the point $(0,7,-7)$.

(b) Find the equation of the plane which passes through the point $(2,3,4)$ and is perpendicular to each of the planes $3x - 4y + 5z + 1 = 0$ and $4x - y - 2z = 0$.

(c) Find the equation of the sphere with centre at $(1,1,1)$ and passing through the point $(1,2,5)$.

(d) (i) Find the equation of the sphere S whose end points of its diameter are given by the points $(0,1,0)$ and $(3,-5,2)$.

(ii) Find the equation of the sphere $S' = 0$ which passes through the intersection of the sphere $S \equiv 0$ with the plane $5x - 2y + 4z - 7 = 0$.

(iii) If $S' = 0$ is a great circle, then show that the equation takes the form

$$S' = 11x^2 + 11y^2 + 11z^2 + 7x + 28y + 10z - 111 = 0.$$

**************THE END************-