University of Kelaniya - Sri Lanka
Center for Distance & Continuing Education
Bachelor of Science(General) External
Third year second semester examination - 2019 (2025 March)
(New Syllabus)
Faculty of Science

Applied Mathematics
AMAT 37623 - Introduction to Fluid Dynamics

No.of Questions: Six(06) No.of Pages: Three(03) Time: Two and Half($2\frac{1}{2}$) hrs
Answer Five(05) Questions Only

1. (a) Determine stream lines and path lines for the flow whose velocity field is given by
$$u = \frac{2x}{t+1},\ v = \frac{-y}{t+1},\ w = \frac{z}{t+1}$$
   (b) Using the definition of a fluid particle's acceleration, determine the acceleration for the following flow field
$$\vec{q} = \left\langle Axy^2t, Bx^2yt, Cxyz^2t \right\rangle,$$
   where A, B, C are constant and x, y, z, t are variables.
   (c) If, $u = \dfrac{ax-by}{x^2+y^2}, v = \dfrac{ax+by}{x^2+y^2}, w = 0$, are the velocity components of three dimensional fluid flow;
      (i) Show that velocity is potential kind
      (ii) Find the velocity potential

2. (a) In the usual notation, derive the following equation of continuity for a fluid flow
$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho\vec{q} = 0.$$
   (b) For a incompressible irrotational fluid show that the above equation reduces to Laplace's equation
$$\nabla^2\phi = 0,$$
   where ϕ is the velocity potential.
   (c) Show that the following velocity
$$\vec{q} = \left\langle \frac{(x^2-y^2)z}{(x^2+y^2)^2}, \frac{2xyz}{(x^2+y^2)^2}, \frac{x}{(x^2+y^2)} \right\rangle$$
   is a possible motion of an incompressible fluid flow.

Continued ...

3. (a) In the usual notation, derive the Euler's equation of motion,

$$\frac{d\vec{q}}{dt} = \vec{F} - \frac{1}{p}\nabla p$$

for an incompressible, unsteady fluid flowing under a conservative force.

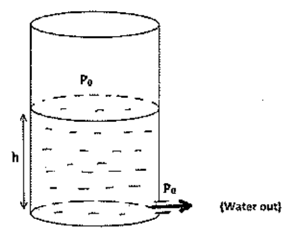
(b) Using part (a) drive the Bernoulli's equation

$$\frac{1}{2}q^2 - \frac{\partial \phi}{\partial t} + \Omega + \int \frac{dP}{\rho} = F(t).$$

(c) Show that at the steady state, equation in part (b) reduces into

$$\frac{1}{2}q^2 + \Omega + \frac{P}{\rho} = \text{constant}.$$

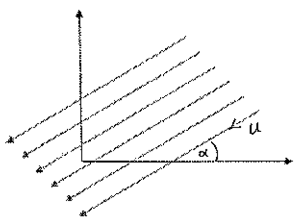
(d) A large open water tank has a small hole at the bottom through which water flows out due to gravity. The water level in the tank is maintained at a constant height h. Assume that the pressure at the top of the water surface is atmospheric pressure P_0 and the velocity of water at the top surface is negligible. Using Bernoulli's equation, derive an expression for the velocity v of the water as it exits the hole.



4. (a) Suppose $z = x + iy$, and the complex potential function $w = f(z) = \varphi(x, y) + i\psi(x, y)$ where φ : velocity potential and, ψ : stream function. If $\frac{dw}{dz}$ is unique through out the region, then w is said to be analytic or regular through out the region. Derive the Cauchy-Riemann Equation.

(b) For a given fluid flow $w = z^2$, discuss the behavior of velocity potential and, stream function.

(c) If the uniform stream is incident to the positive x-axis at angle α, show that the complex potential equal to $uze^{-i\alpha}$.



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5. (a) Let $\varphi(r, \theta, \psi)$ be the velocity potential at any point having spherical polar coordinates (r, θ, ψ) in a field of steady irrotational incompressible flow. State the Laplace's equation using spherical coordinate system.

   (b) If a solid sphere, center o, radius a, moving with uniform velocity $u\hat{k}$, in incompressible fluid of infinite extent which is as rest at infinity. Consider oz is the axis of symmetry and the direction of the unit vector $\hat{k}$.

      (i) Find the velocity potential of the fluid for $r >> a$.

      (ii) Find the kinetic energy of the fluid.

6. (a) In the usual notation, state the complex potential w due to the line source and sink.

   (b) If $w = \dfrac{m}{z - z_1}|\delta z_1|e^{i\alpha}$ in usual notation, Discuss the flow due to a uniform line doublet at O of strength μ per unit length, its axis being along $\overrightarrow{OX}$

   (c) Find the equation of the streamline due to uniform line sources of strength m through the points $A(-1, 0)$, $B(1, 0)$ and a uniform line sink of strength $2m$ through the origin.

   Hint: $\log(x \pm iy) = \dfrac{1}{2}\log(x^2 + y^2) \pm i\ \tan^{-1}(x/y)$

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