



University of Kelaniya - Sri Lanka
Centre for Distance and Continuing Education
Bachelor of Science (External) Degree Examination
Academic Year 2020/2021- SEMESTER I
APPLIED MATHEMATICS
AMAT 36603 - Mathematics for Finance I

September 2024

Time Allowed: 2.5 hours

INSTRUCTIONS

1. This paper contains SEVEN (7) questions and comprises FOUR (4) pages.
 2. Answer only FIVE (5) questions.
 3. All questions carry equal marks.
 4. Scientific calculators are allowed.
 5. Unless otherwise mentioned assume compound interest throughout.
 6. Use correct currency symbols and units of measurements where applicable.
 7. Unless otherwise specified all options are European style.
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1. (a) Determine the nominal interest rate compounded quarterly $i^{(4)}$ that is equivalent to the nominal discount rate compounded semi-annually $d^{(2)}$.

(b) Olivia is to receive payments of X in 2 years and Y in 5 years.
Aidan is to receive payments of $2X$ in 2 years and $5Y$ in 5 years.
Using an annual effective interest rate of 6%, the present value of Olivia's payments is 460, and the present value of Aidan's payments is 2025.
Determine X .

(c) Using an interest rate of i compounded monthly, a payment of \$5000 at the end of two years together with a payment of \$10,000 at the end of four years have a total present value of \$9375.
Using the same interest rate, a deposit of \$27,000 accumulates to Y after six years.
Determine Y .

2. (a) An n -year annuity immediate with annual payments of R has a present value of 10,000.
Given $(1 + i)^n = 2$, determine the present value of a $2n$ -year annuity immediate with annual payments of $2R$, using the same interest rate.

(b) A perpetuity due with annual payments has a first payment \$100 and each subsequent payment is 4% more than its preceding payment.
Using an annual effective interest rate, i , the present value of the perpetuity is \$3300.
Determine i .

(c) A 20-year annuity due has annual payments of 5 for the first 5 years and 15 for the next 15 years.
Determine the accumulated value of the annuity using an annual effective interest rate of 4% for the first 5 years and 5% thereafter.

3. (a) A loan of \$500,000 is repaid with monthly deposits of \$10,000 for as long as necessary, plus a final smaller deposit of \$ X , payable one month after the last regular payment of \$10,000.
Interest on the loan is charged at a nominal rate of 9%, compounded monthly.
Determine X .

(b) Joe repays a loan of \$10,000 by establishing a sinking fund and making 20 equal payments at the end of each year.
The sinking fund earns 7% effective annually.
Immediately after the fifth payment, the yield on the sinking fund increases to 8% effective annually.

At that time, Joe adjusts his sinking fund payment to X so that the sinking fund will accumulate to \$10,000 exactly 20 years after the original loan date.

Determine X .

- (c) A \$1000 par value n -year bond maturing at par with annual coupons of \$100 is purchased for \$1125.

The present value of the redemption value is \$500.

Find n .

4. (a)
- Project P requires an investment of \$4000 at time 0.
The investment pays \$2000 at time 1 and \$4000 at time 2.
 - Project Q requires an investment of X at time 2.
The investment pays \$2000 at time 0 and \$4000 at time 1.
 - Using the net present value method and an interest rate of 10%, the net present values of the two projects are equal.

Calculate X .

- (b) You are given the following table of interest rates:

Calendar Year of Investment	Investment Year Rates			Portfolio Rates
	i_1^Y	i_2^Y	i_3^Y	
Y				i^{Y+3}
2008	0.07	0.06	0.05	0.04
2009	0.06	0.06	0.06	0.06
2010	0.10	0.08	0.08	0.04

\$1000 is invested at the beginning of each of years 2008, 2009, and 2010.

Determine the amount of interest paid for year 2013.

- (c) The relevant characteristics of two bonds forming a portfolio are:
- Bond A has a Macaulay duration of 5 years and a purchase price of X .
 - Bond B has a Macaulay duration of 15 years and a purchase price of $1.5X$.
 - Annual effective interest rate is 10%.

Determine the Modified duration of the portfolio.

5. (a) Define a financial derivative.
- (b) Describe the types of financial derivatives.
- (c) Explain the uses of derivatives.
- (d) What is the difference between the over-the-counter market and the exchange-traded market?
- (e) What is hedging? Briefly describe using appropriate examples.
- (f) What are stock options?
- (g) Explain the relationship between futures prices, forward prices and spot prices.
6. (a) What is the difference between a long forward position and a short forward position?
- (b) Explain the difference between hedging, speculation, and arbitrage.
- (c) What are the most important aspects of the design of a new future contract?
- (d) A trader buys two long July futures contracts on orange juice. Each contract is delivery of 15000 pounds. The current future price is 160 cents per pound, the initial margin is \$6000 per contract, and the maintenance margin is \$4500 per contract. What price change would lead to a margin call? Under what circumstances could \$2000 be withdrawn from the margin account?
7. (a) Explain carefully the difference between selling a call option and buying a put option.
- (b) Suppose XYZ stock has a price of \$50 and pays no dividends. The effective annual interest is 10%. Draw payoff and profit diagrams for a long position in the stock. Verify that profit is 0 at a price in 1 year of \$55.
- (c) It is May, and a trader writes a September call option with a strike price of \$20. The stock price is \$18, and the option price is \$2. Describe the trader's cash flows if the option is held until September and the stock price is \$25 at that time.
- (d) A trader writes a December put option with a strike price of \$30. The price of the option is \$4. Under what circumstances does the trader make a gain?