UNIVERSITY OF KELANIYA – SRI LANKA
FACULTY OF SCIENCE
Bachelor of Science Degree Examination - September 2024
2019 Academic Year – Semester I
STATISTICS
STAT 36522 – Nonparametric Statistics (R)

No. of Questions: Four (04) **No. of Pages: Four (04)** **Time: Two (02) hours**

Answer **all four (04)** questions.
Non – programmable calculators are allowed.

01.

(a) Write two advantages of using non-parametric tests.

(b) Write two disadvantages of using non-parametric tests.

(c) A nutritionist is evaluating the median daily water intake (in ounces) of a group of individuals following a new hydration plan. The hydration plan claims that the median daily water intake should be at least 26 ounces for effectiveness. However, the nutritionist suspects that the actual intake may be less than 26 ounces. To investigate, she collects data from a random sample of 10 participants, and records their daily water intake (in ounces) as follows in Table 1,

Table 1: Daily Water Intake (in ounces)

24.5	28.9	30.7	25.2	27.4
33.3	28.6	26	31.0	32.6

Assuming the data are not normally distributed:

(i) State the null and alternative hypotheses to test the nutritionist's suspicion.

(ii) Identify an appropriate statistical test for this data.

(iii) Using a significance level of $\alpha = 0.05$, determine whether the nutritionist's belief that the actual intake may be less than 26 ounces is supported by the data.

02.

(a) A vocational training institute claims that its current online course helps 55% of its students secure a job within three months of completion. To assess the effectiveness of a new, revised version of the course, the institute conducts a trial with 150 recent graduates. After three months, 90 of the 150 graduates from the revised course report that they have successfully secured employment.

(i) Find a point estimate of population proportion.

(ii) Construct a 95% confidence interval of population proportion.

(iii) Is there sufficient evidence to conclude that the new course shows a statistically significant improvement over the existing 55% success rate? Use a significance level of $\alpha = 0.05$.

(b) A research student has collected GPAs from 12 undergraduates using a simple random sampling technique. The GPAs are given in Table 2,

Table 2: GPAs of students

3.15	3.46	3.21	3.31	3.78	3.60	2.98	3.06	3.33	3.02	3.86	3.24

Test, at the 0.05 level of significance, whether the sequence of GPAs is random.

03.

(a) A tech company is testing the battery endurance of three different models of wireless earbuds — Model X, Model Y, and Model Z. To evaluate performance, the company measures how many hours each earbud can continuously play music before needing to recharge. Each model is tested on 5 randomly selected units. The results (in hours) are as follows in Table 5,

Table 3: battery endurance (in hours)

Model X	Model Y	Model Z
10	9	12
12	10	10
11	12	14
13	11	13
10	9	11

At the 0.05 level of significance, test whether the median battery endurance (in hours) differs among the three earbud models.

(b) A sports science institute is studying the relationship between the training intensity of athletes (measured on a scale from 1–100) and their muscle recovery time (in hours) after a workout. The data from 10 athletes are shown in Table 6,

Table 4: training intensity of athletes and their muscle recovery time

Employee	Training Intensity	Muscle Recovery Time (Hours)
1	75	15
2	90	22
3	80	12
4	45	30
5	60	8
6	55	25
7	50	20
8	70	18
9	65	28
10	85	10

(i) Using Spearman's rank correlation, analyze whether there is a statistically significant relationship between training intensity and recovery time.

(ii) Interpret the correlation between training intensity and recovery time in this group of athletes.

04.

(a) Compare and contrast the Mann-Whitney U test and the Wilcoxon Singed Ranks test.

(b) A fitness research center is comparing the effectiveness of two different diet plans one more affordable and the other more expensive, on weight loss performance. Participants were randomly assigned to follow either of the two plans for one month, under similar exercise and lifestyle conditions.

After the month, the researchers measured the amount of weight lost (in kilograms) by each participant. Ten individuals followed the cheaper diet plan, and ten followed the more expensive plan. The weight loss results are shown below in Table 3:

Table 5: Weight Loss Results (in kilograms)

Cheaper Diet Plan (Group 1)	More Expensive Diet Plan (Group 2)
2.8	3.5
3.1	3.8
3.4	3.6
3.0	3.7
2.7	3.4
2.9	3.0
3.2	3.3
2.6	3.2
3.2	2.9
3.0	2.8

Using an appropriate test, clarify whether there is a statistically significant difference in effectiveness between the two diet plans at 5% level of significance.

(c) An educational psychologist is evaluating the effectiveness of a new study technique designed to improve students' performance on standardized reading comprehension tests. Ten students are selected, and their test scores (out of a standard scale) are recorded before and after completing an 8-week training program based on the new technique. The dataset shown in Table 4,

Table 6: Reading Comprehension Test Scores

Participant	Before Training	After Training
1	210	200
2	220	215
3	205	210
4	225	210
5	210	200
6	235	236
7	240	235
8	230	220
9	220	223
10	250	245

Using an appropriate test, clarify whether the new technique leads to a statistically significant change in reading comprehension scores at 5% level of significance.

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Appendix

Mann-Whitney U Test

$$U_1 = n_1 n_2 + \frac{n_1(n_1+1)}{2} - R_1$$

$$U_2 = n_1 n_2 + \frac{n_2(n_2+1)}{2} - R_2$$

R_1 = Sum of the ranks for group 1, R_2 = Sum of the ranks for group 2, n_1 = Sample Size of 1st sample, n_2 = Sample size of 2nd sample.

Wald-Wolfowitz two sample runs test

For even number of runs

$$P(R = 2k) = \frac{2\binom{n_1-1}{k-1}\binom{n_2-1}{k-1}}{\binom{n_1+n_2}{n_1}}$$

For odd number of runs

$$P(R = 2k+1) = \frac{\binom{n_1-1}{k}\binom{n_2-1}{k-1} + \binom{n_2-1}{k}\binom{n_1-1}{k-1}}{\binom{n_1+n_2}{n_1}}$$

R = Number of Runs, n_1 = Sample Size of 1st sample, n_2 = Sample size of 2nd sample.

Kruskal–Wallis H-test

$$H = \frac{12}{N\,(N+1)} \sum_{i=1}^{k} \frac{T_i^2}{N_i} - 3(N+1)$$

T_i = the rank sum of ith sample, N_i = sample size for ith sample, N = Full sample size

Spearman Rank Correlation

$$\rho = 1 - \frac{6\sum d_i^2}{n\,(n^2-1)}$$

d_i = the difference between the ranks of corresponding variables.
n = sample size

One Sample Sign Test (Large Sample Approximation)

$$Z = \frac{\left(S + \frac{1}{2}\right) - \frac{1}{2}n}{\frac{1}{2}\sqrt{n}}$$

$S = \min(N^-, N^+)$, N^+ = Number of positive differences, N^- = Number of negative differences, n = sample size

Wilcoxon Sign Rank Test (Large Sample Approximation)

$$Z = \frac{W - \frac{n(n+1)}{4}}{\sqrt{\frac{n(n+1)(2n+1)}{24}}}$$

$W = \min(W^-, W^+)$, W^+ = the sum of the ranks with positive signs, W^- = the sum of the ranks with negative signs, n = sample size

Large Sample Confidence Interval of Population Proportion

$$\hat{p} - Z_{1-\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} < p < \hat{p} + Z_{1-\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$\hat{p}$ = proportion of the sample with the characteristic of interest
n = sample size

n	x	$p = 0.05$	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
2	0	0.9025	0.8100	0.7225	0.6400	0.5625	0.4900	0.4225	0.3600	0.3025	0.2500
	1	0.9975	0.9900	0.9775	0.9600	0.9375	0.9100	0.8775	0.8400	0.7975	0.7500
3	0	0.8574	0.7290	0.6141	0.5120	0.4219	0.3430	0.2746	0.2160	0.1664	0.1250
	1	0.9928	0.9720	0.9393	0.8960	0.8438	0.7840	0.7183	0.6480	0.5748	0.5000
	2	0.9999	0.9990	0.9966	0.9920	0.9844	0.9730	0.9571	0.9360	0.9089	0.8750
4	0	0.8145	0.6561	0.5220	0.4096	0.3164	0.2401	0.1785	0.1296	0.0915	0.0625
	1	0.9860	0.9477	0.8905	0.8192	0.7383	0.6517	0.5630	0.4752	0.3910	0.3125
	2	0.9995	0.9963	0.9880	0.9728	0.9492	0.9163	0.8735	0.8208	0.7585	0.6875
	3	1.0000	0.9999	0.9995	0.9984	0.9961	0.9919	0.9850	0.9744	0.9590	0.9375
5	0	0.7738	0.5905	0.4437	0.3277	0.2373	0.1681	0.1160	0.0778	0.0503	0.0313
	1	0.9774	0.9185	0.8352	0.7373	0.6328	0.5282	0.4284	0.3370	0.2562	0.1875
	2	0.9988	0.9914	0.9734	0.9421	0.8965	0.8369	0.7648	0.6826	0.5931	0.5000
	3	1.0000	0.9995	0.9978	0.9933	0.9844	0.9692	0.9460	0.9130	0.8688	0.8125
	4	1.0000	1.0000	0.9999	0.9997	0.9990	0.9976	0.9947	0.9898	0.9815	0.9688
6	0	0.7351	0.5314	0.3771	0.2621	0.1780	0.1176	0.0754	0.0467	0.0277	0.0156
	1	0.9672	0.8857	0.7765	0.6554	0.5339	0.4202	0.3191	0.2333	0.1636	0.1094
	2	0.9978	0.9842	0.9527	0.9011	0.8306	0.7443	0.6471	0.5443	0.4415	0.3438
	3	0.9999	0.9987	0.9941	0.9830	0.9624	0.9295	0.8826	0.8208	0.7447	0.6563
	4	1.0000	0.9999	0.9996	0.9984	0.9954	0.9891	0.9777	0.9590	0.9308	0.8906
	5	1.0000	1.0000	1.0000	0.9999	0.9998	0.9993	0.9982	0.9959	0.9917	0.9844
7	0	0.6983	0.4783	0.3206	0.2097	0.1335	0.0824	0.0490	0.0280	0.0152	0.0078
	1	0.9556	0.8503	0.7166	0.5767	0.4449	0.3294	0.2338	0.1586	0.1024	0.0625
	2	0.9962	0.9743	0.9262	0.8520	0.7564	0.6471	0.5323	0.4199	0.3164	0.2266
	3	0.9998	0.9973	0.9879	0.9667	0.9294	0.8740	0.8002	0.7102	0.6083	0.5000
	4	1.0000	0.9998	0.9988	0.9953	0.9871	0.9712	0.9444	0.9037	0.8471	0.7734
	5	1.0000	1.0000	0.9999	0.9996	0.9987	0.9962	0.9910	0.9812	0.9643	0.9375
	6	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9994	0.9984	0.9963	0.9922
8	0	0.6634	0.4305	0.2725	0.1678	0.1001	0.0576	0.0319	0.0168	0.0084	0.0039
	1	0.9428	0.8131	0.6572	0.5033	0.3671	0.2553	0.1691	0.1064	0.0632	0.0352
	2	0.9942	0.9619	0.8948	0.7969	0.6785	0.5518	0.4278	0.3154	0.2201	0.1445
	3	0.9996	0.9950	0.9786	0.9437	0.8862	0.8059	0.7064	0.5941	0.4770	0.3633
	4	1.0000	0.9996	0.9971	0.9896	0.9727	0.9420	0.8939	0.8263	0.7396	0.6367
	5	1.0000	1.0000	0.9998	0.9988	0.9958	0.9887	0.9747	0.9502	0.9115	0.8555
	6	1.0000	1.0000	1.0000	0.9999	0.9996	0.9987	0.9964	0.9915	0.9819	0.9648
	7	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9993	0.9983	0.9961
9	0	0.6302	0.3874	0.2316	0.1342	0.0751	0.0404	0.0207	0.0101	0.0046	0.0020
	1	0.9288	0.7748	0.5995	0.4362	0.3003	0.1960	0.1211	0.0705	0.0385	0.0195
	2	0.9916	0.9470	0.8591	0.7382	0.6007	0.4628	0.3373	0.2318	0.1495	0.0898
	3	0.9994	0.9917	0.9661	0.9144	0.8343	0.7297	0.6089	0.4826	0.3614	0.2539
	4	1.0000	0.9991	0.9944	0.9804	0.9511	0.9012	0.8283	0.7334	0.6214	0.5000
	5	1.0000	0.9999	0.9994	0.9969	0.9900	0.9747	0.9464	0.9006	0.8342	0.7461
	6	1.0000	1.0000	1.0000	0.9997	0.9987	0.9957	0.9888	0.9750	0.9502	0.9102
	7	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9986	0.9962	0.9909	0.9805
	8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9992	0.9980

n	x	p = 0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
10	0	0.5987	0.3487	0.1969	0.1074	0.0563	0.0282	0.0135	0.0060	0.0025	0.0010
	1	0.9139	0.7361	0.5443	0.3758	0.2440	0.1493	0.0860	0.0464	0.0233	0.0107
	2	0.9885	0.9298	0.8202	0.6778	0.5256	0.3828	0.2616	0.1673	0.0996	0.0547
	3	0.9990	0.9872	0.9500	0.8791	0.7759	0.6496	0.5138	0.3823	0.2660	0.1719
	4	0.9999	0.9984	0.9901	0.9672	0.9219	0.8497	0.7515	0.6331	0.5044	0.3770
	5	1.0000	0.9999	0.9986	0.9936	0.9803	0.9527	0.9051	0.8338	0.7384	0.6230
	6	1.0000	1.0000	0.9999	0.9991	0.9965	0.9894	0.9740	0.9452	0.8980	0.8281
	7	1.0000	1.0000	1.0000	0.9999	0.9996	0.9984	0.9952	0.9877	0.9726	0.9453
	8	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995	0.9983	0.9955	0.9893
	9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9990
11	0	0.5688	0.3138	0.1673	0.0859	0.0422	0.0198	0.0088	0.0036	0.0014	0.0005
	1	0.8981	0.6974	0.4922	0.3221	0.1971	0.1130	0.0606	0.0302	0.0139	0.0059
	2	0.9848	0.9104	0.7788	0.6174	0.4552	0.3127	0.2001	0.1189	0.0652	0.0327
	3	0.9984	0.9815	0.9306	0.8389	0.7133	0.5696	0.4256	0.2963	0.1911	0.1133
	4	0.9999	0.9972	0.9841	0.9496	0.8854	0.7897	0.6683	0.5328	0.3971	0.2744
	5	1.0000	0.9997	0.9973	0.9883	0.9657	0.9218	0.8513	0.7535	0.6331	0.5000
	6	1.0000	1.0000	0.9997	0.9980	0.9924	0.9784	0.9499	0.9006	0.8262	0.7256
	7	1.0000	1.0000	1.0000	0.9998	0.9988	0.9957	0.9878	0.9707	0.9390	0.8867
	8	1.0000	1.0000	1.0000	1.0000	0.9999	0.9994	0.9980	0.9941	0.9852	0.9673
	9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9993	0.9978	0.9941
	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9995
12	0	0.5404	0.2824	0.1422	0.0687	0.0317	0.0138	0.0057	0.0022	0.0008	0.0002
	1	0.8816	0.6590	0.4435	0.2749	0.1584	0.0850	0.0424	0.0196	0.0083	0.0032
	2	0.9804	0.8891	0.7358	0.5583	0.3907	0.2528	0.1513	0.0834	0.0421	0.0193
	3	0.9978	0.9744	0.9078	0.7946	0.6488	0.4925	0.3467	0.2253	0.1345	0.0730
	4	0.9998	0.9957	0.9761	0.9274	0.8424	0.7237	0.5833	0.4382	0.3044	0.1938
	5	1.0000	0.9995	0.9954	0.9806	0.9456	0.8822	0.7873	0.6652	0.5269	0.3872
	6	1.0000	0.9999	0.9993	0.9961	0.9857	0.9614	0.9154	0.8418	0.7393	0.6128
	7	1.0000	1.0000	0.9999	0.9994	0.9972	0.9905	0.9745	0.9427	0.8883	0.8062
	8	1.0000	1.0000	1.0000	0.9999	0.9996	0.9983	0.9944	0.9847	0.9644	0.9270
	9	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9992	0.9972	0.9921	0.9807
	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9989	0.9968
	11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998
13	0	0.5133	0.2542	0.1209	0.0550	0.0238	0.0097	0.0037	0.0013	0.0004	0.0001
	1	0.8646	0.6213	0.3983	0.2336	0.1267	0.0637	0.0296	0.0126	0.0049	0.0017
	2	0.9755	0.8661	0.6920	0.5017	0.3326	0.2025	0.1132	0.0579	0.0269	0.0112
	3	0.9969	0.9658	0.8820	0.7473	0.5843	0.4206	0.2783	0.1686	0.0929	0.0461
	4	0.9997	0.9935	0.9658	0.9009	0.7940	0.6543	0.5005	0.3530	0.2279	0.1334
	5	1.0000	0.9991	0.9925	0.9700	0.9198	0.8346	0.7159	0.5744	0.4268	0.2905
	6	1.0000	0.9999	0.9987	0.9930	0.9757	0.9376	0.8705	0.7712	0.6437	0.5000
	7	1.0000	1.0000	0.9998	0.9988	0.9944	0.9818	0.9538	0.9023	0.8212	0.7095
	8	1.0000	1.0000	1.0000	0.9998	0.9990	0.9960	0.9874	0.9679	0.9302	0.8666
	9	1.0000	1.0000	1.0000	1.0000	0.9999	0.9993	0.9975	0.9922	0.9797	0.9539
	10	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9987	0.9959	0.9888
	11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995	0.9983
	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999

Critical Values of the Wilcoxon Signed Ranks Test

n	Two-Tailed Test		One-Tailed Test	
	α = .05	α = .01	α = .05	α = .01
5	--	--	0	--
6	0	--	2	--
7	2	--	3	0
8	3	0	5	1
9	5	1	8	3
10	8	3	10	5
11	10	5	13	7
12	13	7	17	9
13	17	9	21	12
14	21	12	25	15
15	25	15	30	19
16	29	19	35	23
17	34	23	41	27
18	40	27	47	32
19	46	32	53	37
20	52	37	60	43
21	58	42	67	49
22	65	48	75	55
23	73	54	83	62
24	81	61	91	69
25	89	68	100	76
26	98	75	110	84
27	107	83	119	92
28	116	91	130	101
29	126	100	140	110
30	137	109	151	120

Critical Values of r in the runs test

Given in the tables are various critical values of r for values of m and n less than or equal to 20. For the one-sample runs test, any observed value of r which is less than or equal to the smaller value, or is greater than or equal to the larger value in a pair is significant at the $\alpha = .05$ level.

m \ n	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2											2 –	2 –	2 –	2 –	2 –	2 –	2 –	2 –	2 –
3					2 –	2 –	2 –	2 –	2 –	2 –	2 –	2 –	2 –	3 –	3 –	3 –	3 –	3 –	3 –
4				2 9	2 9	2 –	3 –	3 –	3 –	3 –	3 –	3 –	3 –	3 –	4 –	4 –	4 –	4 –	4 –
5			2 9	2 10	3 10	3 11	3 11	3 –	3 –	4 –	4 –	4 –	4 –	4 –	4 –	4 –	5 –	5 –	5 –
6		2 –	2 9	3 10	3 11	3 12	3 12	4 13	4 13	4 13	4 13	5 –	5 –	5 –	5 –	5 –	5 –	6 –	6 –
7		2 –	2 –	3 11	3 12	3 13	4 13	4 14	5 14	5 14	5 14	5 15	5 15	6 15	6 –	6 –	6 –	6 –	6 –
8		2 –	3 –	3 11	3 12	4 13	4 14	5 14	5 15	5 15	6 16	6 16	6 16	6 16	6 17	7 17	7 17	7 17	7 17
9		2 –	3 –	3 –	4 13	4 14	5 14	5 15	5 16	6 16	6 16	6 17	7 17	7 18	7 18	7 18	8 18	8 18	8 18
10		2 –	3 –	3 –	4 13	5 14	5 15	5 16	6 16	6 17	7 17	7 18	7 18	7 18	8 19	8 19	8 19	8 20	9 20
11		2 –	3 –	4 –	4 13	5 14	5 15	6 16	6 17	7 17	7 18	7 19	8 19	8 19	8 20	9 20	9 20	9 21	9 21
12	2 –	2 –	3 –	4 –	4 13	5 14	6 16	6 16	7 17	7 18	7 19	8 19	8 20	8 20	9 21	9 21	9 21	10 22	10 22
13	2 –	2 –	3 –	4 –	5 –	5 15	6 16	6 17	7 18	7 19	8 19	8 20	9 20	9 21	9 21	10 22	10 22	10 23	10 23
14	2 –	2 –	3 –	4 –	5 –	5 15	6 16	7 17	7 18	8 19	8 20	9 20	9 21	9 22	10 22	10 23	10 23	11 23	11 24
15	2 –	3 –	3 –	4 –	5 –	6 15	6 16	7 18	7 18	8 19	8 20	9 21	9 22	10 22	10 23	11 23	11 24	11 24	12 25
16	2 –	3 –	4 –	4 –	5 –	6 –	6 17	7 18	8 19	8 20	9 21	9 21	10 22	10 23	11 23	11 24	11 25	12 25	12 25
17	2 –	3 –	4 –	4 –	5 –	6 –	7 17	7 18	8 19	9 20	9 21	10 22	10 23	11 23	11 24	11 25	12 25	12 26	13 26
18	2 –	3 –	4 –	5 –	5 –	6 –	7 17	8 18	8 19	9 20	9 21	10 22	10 23	11 24	11 25	12 25	12 26	13 26	13 27
19	2 –	3 –	4 –	5 –	6 –	6 –	7 17	8 18	8 20	9 21	10 22	10 23	11 23	11 24	12 25	12 26	13 26	13 27	13 27
20	2 –	3 –	4 –	5 –	6 –	6 –	7 17	8 18	9 20	9 21	10 22	10 23	11 24	12 25	12 25	13 26	13 27	13 27	14 28

* Adapted from Swed, and Eisenhart, C. (1943). Tables for testing randomness of grouping in a sequence of alternatives. *Annals of Mathematical Statistics*, **14**, 83–86, with the kind permission of the authors and publisher.

Critical Values of the Mann – Whitney U

n_2	α	n_1 = 3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
3	.05	--	0	0	1	1	2	2	3	3	4	4	5	5	6	6	7	7	8
	.01	--	0	0	0	0	0	0	0	0	1	1	1	2	2	2	2	3	3
4	.05	--	0	1	2	3	4	4	5	6	7	8	9	10	11	11	12	13	14
	.01	--	--	0	0	0	1	1	2	2	3	3	4	5	5	6	6	7	8
5	.05	0	1	2	3	5	6	7	8	9	11	12	13	14	15	17	18	19	20
	.01	--	--	0	1	1	2	3	4	5	6	7	7	8	9	10	11	12	13
6	.05	1	2	3	5	6	8	10	11	13	14	16	17	19	21	22	24	25	27
	.01	--	0	1	2	3	4	5	6	7	9	10	11	12	13	15	16	17	18
7	.05	1	3	5	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34
	.01	--	0	1	3	4	6	7	9	10	12	13	15	16	18	19	21	22	24
8	.05	2	4	6	8	10	13	15	17	19	22	24	26	29	31	34	36	38	41
	.01	--	1	2	4	6	7	9	11	13	15	17	18	20	22	24	26	28	30
9	.05	2	4	7	10	12	15	17	20	23	26	28	31	34	37	39	42	45	48
	.01	0	1	3	5	7	9	11	13	16	18	20	22	24	27	29	31	33	36
10	.05	3	5	8	11	14	17	20	23	26	29	33	36	39	42	45	48	52	55
	.01	0	2	4	6	9	11	13	16	18	21	24	26	29	31	34	37	39	42
11	.05	3	6	9	13	16	19	23	26	30	33	37	40	44	47	51	55	58	62
	.01	0	2	5	7	10	13	16	18	21	24	27	30	33	36	39	42	45	48
12	.05	4	7	11	14	18	22	26	29	33	37	41	45	49	53	57	61	65	69
	.01	1	3	6	9	12	15	18	21	24	27	31	34	37	41	44	47	51	54
13	.05	4	8	12	16	20	24	28	33	37	41	45	50	54	59	63	67	72	76
	.01	1	3	7	10	13	17	20	24	27	31	34	38	42	45	49	53	56	60
14	.05	5	9	13	17	22	26	31	36	40	45	50	55	59	64	67	74	78	83
	.01	1	4	7	11	15	18	22	26	30	34	38	42	46	50	54	58	63	67
15	.05	5	10	14	19	24	29	34	39	44	49	54	59	64	70	75	80	85	90
	.01	2	5	8	12	16	20	24	29	33	37	42	46	51	55	60	64	69	73
16	.05	6	11	15	21	26	31	37	42	47	53	59	64	70	75	81	86	92	98
	.01	2	5	9	13	18	22	27	31	36	41	45	50	55	60	65	70	74	79
17	.05	6	11	17	22	28	34	39	45	51	57	63	67	75	81	87	93	99	105
	.01	2	6	10	15	19	24	29	34	39	44	49	54	60	65	70	75	81	86
18	.05	7	12	18	24	30	36	42	48	55	61	67	74	80	86	93	99	106	112
	.01	2	6	11	16	21	26	31	37	42	47	53	58	64	70	75	81	87	92
19	.05	7	13	19	25	32	38	45	52	58	65	72	78	85	92	99	106	113	119
	.01	3	7	12	17	22	28	33	39	45	51	56	63	69	74	81	87	93	99
20	.05	8	14	20	27	34	41	48	55	62	69	76	83	90	98	105	112	119	127
	.01	3	8	13	18	24	30	36	42	48	54	60	67	73	79	86	92	99	105

Chi-square Distribution Table

d.f.	.995	.99	.975	.95	.9	.1	.05	.025	.01
1	0.00	0.00	0.00	0.00	0.02	2.71	3.84	5.02	6.63
2	0.01	0.02	0.05	0.10	0.21	4.61	5.99	7.38	9.21
3	0.07	0.11	0.22	0.35	0.58	6.25	7.81	9.35	11.34
4	0.21	0.30	0.48	0.71	1.06	7.78	9.49	11.14	13.28
5	0.41	0.55	0.83	1.15	1.61	9.24	11.07	12.83	15.09
6	0.68	0.87	1.24	1.64	2.20	10.64	12.59	14.45	16.81
7	0.99	1.24	1.69	2.17	2.83	12.02	14.07	16.01	18.48
8	1.34	1.65	2.18	2.73	3.49	13.36	15.51	17.53	20.09
9	1.73	2.09	2.70	3.33	4.17	14.68	16.92	19.02	21.67
10	2.16	2.56	3.25	3.94	4.87	15.99	18.31	20.48	23.21
11	2.60	3.05	3.82	4.57	5.58	17.28	19.68	21.92	24.72
12	3.07	3.57	4.40	5.23	6.30	18.55	21.03	23.34	26.22
13	3.57	4.11	5.01	5.89	7.04	19.81	22.36	24.74	27.69
14	4.07	4.66	5.63	6.57	7.79	21.06	23.68	26.12	29.14
15	4.60	5.23	6.26	7.26	8.55	22.31	25.00	27.49	30.58
16	5.14	5.81	6.91	7.96	9.31	23.54	26.30	28.85	32.00
17	5.70	6.41	7.56	8.67	10.09	24.77	27.59	30.19	33.41
18	6.26	7.01	8.23	9.39	10.86	25.99	28.87	31.53	34.81
19	6.84	7.63	8.91	10.12	11.65	27.20	30.14	32.85	36.19
20	7.43	8.26	9.59	10.85	12.44	28.41	31.41	34.17	37.57
22	8.64	9.54	10.98	12.34	14.04	30.81	33.92	36.78	40.29
24	9.89	10.86	12.40	13.85	15.66	33.20	36.42	39.36	42.98
26	11.16	12.20	13.84	15.38	17.29	35.56	38.89	41.92	45.64
28	12.46	13.56	15.31	16.93	18.94	37.92	41.34	44.46	48.28
30	13.79	14.95	16.79	18.49	20.60	40.26	43.77	46.98	50.89
32	15.13	16.36	18.29	20.07	22.27	42.58	46.19	49.48	53.49
34	16.50	17.79	19.81	21.66	23.95	44.90	48.60	51.97	56.06
38	19.29	20.69	22.88	24.88	27.34	49.51	53.38	56.90	61.16
42	22.14	23.65	26.00	28.14	30.77	54.09	58.12	61.78	66.21
46	25.04	26.66	29.16	31.44	34.22	58.64	62.83	66.62	71.20
50	27.99	29.71	32.36	34.76	37.69	63.17	67.50	71.42	76.15
55	31.73	33.57	36.40	38.96	42.06	68.80	73.31	77.38	82.29
60	35.53	37.48	40.48	43.19	46.46	74.40	79.08	83.30	88.38
65	39.38	41.44	44.60	47.45	50.88	79.97	84.82	89.18	94.42
70	43.28	45.44	48.76	51.74	55.33	85.53	90.53	95.02	100.43
75	47.21	49.48	52.94	56.05	59.79	91.06	96.22	100.84	106.39
80	51.17	53.54	57.15	60.39	64.28	96.58	101.88	106.63	112.33
85	55.17	57.63	61.39	64.75	68.78	102.08	107.52	112.39	118.24
90	59.20	61.75	65.65	69.13	73.29	107.57	113.15	118.14	124.12
95	63.25	65.90	69.92	73.52	77.82	113.04	118.75	123.86	129.97
100	67.33	70.06	74.22	77.93	82.36	118.50	124.34	129.56	135.81