**University of Kelaniya – Sri Lanka**
**Centre for Distance & Continuing Education**
**Bachelor of Science (General) External**
**Third year second semester examination - 2019 (2025 February/March)**
**(New Syllabus)**
**Faculty of Science**

**Statistics**
**STAT 37532– Statistical Quality Control**

No. of Questions: 04 No. of Pages: 05(Five) Time: **Two & half (2 1/2) Hours.**

Answer All **Four (04)** questions.

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1. *(a)*. A factory produces metal rods with a length that follows an unknown distribution with a mean of 50 cm and a standard deviation of 5 cm. A quality control engineer takes **random samples of size 40** from the production line and calculates the sample mean length.
   *(i)* According to the **Central Limit Theorem**, what is the expected shape of the distribution of the sample means?
   *(ii)* What will be the mean and standard deviation of the sample mean distribution?
   *(iii)* Calculate the probability that a randomly selected sample of 40 rods has an average length greater than 51 cm.

   *(b)* What are the advantages of utilizing control charts in quality management and process monitoring?

   *(c)* A cable harness assembly shop is experiencing bonding failures during the curing process. Identify **five possible causes** using Ishikawa's major categories.

   *(d)* A company monitors a **slow-moving production process** using Moving Average (MA) and Moving Range (MR) charts.
   (i) Explain how MA and MR charts help detect small shifts in the process and how they reduce noise in data.
   (ii) Discuss whether smaller or larger subgroups are more effective for these charts and why.

2. (a) If a process is monitored using an $\bar{X}$ control chart, examine how a change in sample size from n = 12 to n = 8 impacts the calculation of control limits and may lead to an "out-of-control" signal. (Standard tables are attached.)

Hint: The **Upper Control Limit (UCL)** and **Lower Control Limit (LCL)** for an $\bar{X}$ chart are calculated as

$$UCL = \bar{X} + A_2 \times \bar{R}$$
$$LCL = \bar{X} - A_2 \times \bar{R}$$

Where,

$$\bar{X} = Sample\ mean$$
$$\bar{R} = Average\ Range$$

$$A_2 = Control\ chart\ factor$$

(b) Suppose **a chemical manufacturing process** monitors the **purity level** of its product. Due to the nature of the process, only **one observation per time period** is available, making subgroup sampling impractical. Why is an **Individual Control Chart (X-chart) and Moving Range (MR) Chart** preferred over an **$\bar{X}$ and R-chart** in this scenario?

(c)

(i) Explain the difference between process capability (Cpk) and process performance (Ppk).

(ii) A steel rod manufacturing company seeks to determine whether its production process is consistently delivering products within quality standards. The process specifications are as follows:-

- Target Diameter = 50 mm
- Upper Specification Limit (USL) = 52 mm
- Lower Specification Limit (LSL) = 48 mm
- Process Mean (μ) = 50.5 mm
- Process Standard Deviation (σ) = 0.5 mm

Using these parameters, assess the ability of the process to maintain acceptable quality levels. Discuss whether the current process is well-controlled and capable of meeting specifications, and suggest potential improvements if needed.

- **Cp:** process capability index
- **Cpk:** minimum process capability index
- **Pp:** process performance index
- **Ppk:** minimum process performance index

| | |
|---|---|
| $C_p = \frac{(USL - LSL)}{6\hat{\sigma}}$ | $P_p = \frac{(USL - LSL)}{6SD}$ |
| $Cpkl = \frac{Mean - LSL}{3\hat{\sigma}}$ | $Ppkl = \frac{Mean - LSL}{3SD}$ |
| $Cpku = \frac{USL - Mean}{3\hat{\sigma}}$ | $Ppku = \frac{USL - Mean}{3SD}$ |
| $C_{pk}$= min (Cpkl, Cpku) | $P_{pk}$= min (Ppkl, Ppku) |

$$Cpk = \frac{Total\ Tolerance}{Process\ Spread}$$

**USL:** upper specification limit;

**LSL:** lower specification limit;

**Mean:** grand average of all the data

**Sigma hat:** estimated inherent variability (noise) of a stable process

**SD:** overall variability

3. (a) What is the main purpose of the Western Electric Rules in Statistical Quality Control?

   (b) A **quality control team** in a **manufacturing plant** monitors two aspects of production using **P-charts and C-charts.**

   (i) **Defective Bolts (P-Chart)**

   The team inspects **samples of 150 bolts** per batch for defects. Over **40 batches**, the **average proportion of defective bolts** was **0.03.**

   A. **Calculate the control limits** (UCL & LCL) for a **P-chart** at **3σ limits.**

   B. If a new batch contains **8 defective bolts** in a sample of **150**, determine whether the process is in control.

   C. Explain why a **P-chart** is suitable for monitoring defective bolts.

   (ii) **Scratches on Laminated Sheets (C-Chart)**

   The team also monitors laminated sheets for surface defects. Over **35 sheets**, the **average number of scratches per sheet** was **5.5.**

   A. **Calculate the control limits** (UCL & LCL) for a **C-chart** at **3σ limits.**

   B. If a newly inspected sheet contains **12 scratches**, analyze whether this signals an out-of-control process.

   C. Justify why a **C-chart** is appropriate for this type of defect monitoring instead of a P-chart.

4. (a) In the context of Statistical Quality Control (SQC), explain the following:

(i) Type I Error: What happens if the inspector rejects a good batch that actually meets the quality standards?

(ii) Type II Error: What happens if the inspector accepts a defective batch that does not meet the quality standards?

(b) Explain the methodology of a single sampling plan, including the steps involved in determining whether to accept or reject a lot.

(c) A bakery supplies cookies in large batches. To ensure quality, an **acceptance sampling plan** is used as follows: A **random sample of 40 cookies** is taken from each batch.

If **two or fewer defective cookies** (e.g., broken or burnt) are found, the batch is **accepted**.

If **three or more defective cookies** are found, the batch is **rejected**.

(i) If the actual defect rate in a batch is **5%**, what is the probability that the batch will be **accepted**?

(ii) If the actual defect rate in a batch is **12%**, what is the probability that the batch will be **accepted**?

**Factors useful in the Construction of Control Charts**

| | *Chart for Averages* | | | *Chart for standard deviations* | | | | | *Chart for Ranges* | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | *Factors for Control Limits* | | | *Factors for:* *Central Line* | *Control Limits* | | | | *Factors for Central Line* | | *Factors for Control Limits* | | | | |
| n | A | $A_2$ | $A_3$ | $c_4$ | $B_3$ | $B_4$ | $B_5$ | $B_6$ | $d_2$ | $1/d_2$ | $d_3$ | $D_1$ | $D_2$ | $D_3$ | $D_4$ |
| 2 | 2.121 | 1.880 | 2.659 | 0.7979 | 0 | 3.267 | 0 | 2.606 | 1.128 | 0.8862 | 0.852 | 0 | 3.686 | 0 | 3.266 |
| 3 | 1.732 | 1.023 | 1.954 | 0.8862 | 0 | 2.568 | 0 | 2.276 | 1.693 | 0.5908 | 0.888 | 0 | 4.357 | 0 | 2.574 |
| 4 | 1.500 | 0.729 | 1.628 | 0.9213 | 0 | 2.266 | 0 | 2.088 | 2.059 | 0.4857 | 0.879 | 0 | 4.697 | 0 | 2.281 |
| 5 | 1.342 | 0.577 | 1.427 | 0.9400 | 0 | 2.089 | 0 | 1.964 | 2.326 | 0.4299 | 0.864 | 0 | 4.918 | 0 | 2.114 |
| 6 | 1.225 | 0.483 | 1.287 | 0.9515 | 0.030 | 1.970 | 0.029 | 1.874 | 2.534 | 0.3946 | 0.848 | 0 | 5.078 | 0 | 2.003 |
| 7 | 1.134 | 0.419 | 1.182 | 0.9594 | 0.118 | 1.882 | 0.113 | 1.806 | 2.704 | 0.3698 | 0.833 | 0.206 | 5.203 | 0.076 | 1.924 |
| 8 | 1.061 | 0.373 | 1.099 | 0.9650 | 0.185 | 1.815 | 0.179 | 1.751 | 2.847 | 0.3512 | 0.819 | 0.389 | 5.306 | 0.137 | 1.863 |
| 9 | 1.000 | 0.337 | 1.032 | 0.9693 | 0.239 | 1.761 | 0.232 | 1.707 | 2.970 | 0.3367 | 0.807 | 0.548 | 5.392 | 0.184 | 1.816 |
| 10 | 0.949 | 0.308 | 0.975 | 0.9727 | 0.284 | 1.716 | 0.276 | 1.669 | 3.078 | 0.3249 | 0.797 | 0.688 | 5.467 | 0.223 | 1.777 |
| 11 | 0.905 | 0.285 | 0.927 | 0.9754 | 0.321 | 1.679 | 0.313 | 1.637 | 3.173 | 0.3152 | 0.787 | 0.813 | 5.533 | 0.256 | 1.744 |
| 12 | 0.866 | 0.266 | 0.886 | 0.9776 | 0.354 | 1.646 | 0.346 | 1.610 | 3.258 | 0.3069 | 0.778 | 0.924 | 5.593 | 0.284 | 1.716 |
| 13 | 0.832 | 0.249 | 0.850 | 0.9794 | 0.382 | 1.618 | 0.374 | 1.585 | 3.336 | 0.2998 | 0.770 | 1.026 | 5.646 | 0.307 | 1.693 |
| 14 | 0.802 | 0.235 | 0.817 | 0.9810 | 0.406 | 1.594 | 0.399 | 1.563 | 3.407 | 0.2935 | 0.763 | 1.119 | 5.695 | 0.328 | 1.672 |
| 15 | 0.775 | 0.223 | 0.789 | 0.9823 | 0.428 | 1.572 | 0.421 | 1.544 | 3.472 | 0.2880 | 0.756 | 1.204 | 5.739 | 0.347 | 1.653 |
| 16 | 0.750 | 0.212 | 0.763 | 0.9835 | 0.448 | 1.552 | 0.440 | 1.526 | 3.532 | 0.2831 | 0.750 | 1.283 | 5.781 | 0.363 | 1.637 |
| 17 | 0.728 | 0.203 | 0.739 | 0.9845 | 0.466 | 1.534 | 0.458 | 1.511 | 3.588 | 0.2787 | 0.744 | 1.357 | 5.819 | 0.378 | 1.622 |
| 18 | 0.707 | 0.194 | 0.718 | 0.9854 | 0.482 | 1.518 | 0.475 | 1.496 | 3.640 | 0.2747 | 0.738 | 1.425 | 5.855 | 0.392 | 1.608 |
| 19 | 0.688 | 0.187 | 0.698 | 0.9862 | 0.497 | 1.503 | 0.490 | 1.483 | 3.689 | 0.2711 | 0.733 | 1.490 | 5.888 | 0.404 | 1.596 |
| 20 | 0.671 | 0.180 | 0.680 | 0.9869 | 0.510 | 1.490 | 0.504 | 1.470 | 3.735 | 0.2677 | 0.728 | 1.550 | 5.920 | 0.415 | 1.585 |
| 21 | 0.655 | 0.173 | 0.663 | 0.9876 | 0.523 | 1.477 | 0.516 | 1.459 | 3.778 | 0.2647 | 0.724 | 1.607 | 5.950 | 0.425 | 1.575 |
| 22 | 0.640 | 0.167 | 0.647 | 0.9882 | 0.534 | 1.466 | 0.528 | 1.448 | 3.819 | 0.2618 | 0.719 | 1.661 | 5.978 | 0.435 | 1.565 |
| 23 | 0.626 | 0.162 | 0.633 | 0.9887 | 0.545 | 1.455 | 0.539 | 1.438 | 3.858 | 0.2592 | 0.715 | 1.712 | 6.004 | 0.444 | 1.556 |
| 24 | 0.612 | 0.157 | 0.619 | 0.9892 | 0.555 | 1.445 | 0.549 | 1.429 | 3.895 | 0.2567 | 0.712 | 1.761 | 6.030 | 0.452 | 1.548 |
| 25 | 0.600 | 0.153 | 0.606 | 0.9896 | 0.565 | 1.435 | 0.559 | 1.420 | 3.931 | 0.2544 | 0.708 | 1.807 | 6.055 | 0.460 | 1.540 |
| Over 25 | $3/\sqrt{n}$ | $3/d_2\sqrt{n}$ | ... | ... | * | ** | ... | ... | ... | ... | ... | ... | ... | ... | ... |

$* \; 1 - \frac{3}{\sqrt{2n}}$ $\qquad ** \; 1 + \frac{3}{\sqrt{2n}}$