UNIVERSITY OF KELANIYA – SRI LANKA
FACULTY OF SCIENCE
Bachelor of Science Degree Examination - September 2024
2019 Academic Year – Semester I
STATISTICS
STAT 36522 – Nonparametric Statistics

No. of Questions: Four (04)   No. of Pages: Four (04)   Time: Two (02) hours

Answer **all four (04)** questions.
Non – programmable calculators are allowed.

01.

(a) Write two situations where we use nonparametric statistics.

(b) Explain why median is a robust estimator for extreme distributions. Give an example to support your answer.

(c) A car manufacturer claims that the median mileage for their cars is 26 miles per gallon (mpg). However, a quality controller suspects that the actual mileage might be lower than 26 mpg. To test this suspicion, the quality controller takes a sample of 10 cars, with the following mileages in Table 1,

Table 1: Mileages of cars (in mpg)

24.5	28.9	30.7	25.2	27.4
33.3	28.6	26	31.0	32.6

Assuming the data are not normally distributed:

(i) State the null and alternative hypotheses to test the quality controller's suspicion.

(ii) Identify an appropriate statistical test for this data.

(iii) Using a significance level of $\alpha = 0.05$, determine whether the quality controller's belief that the median mileage is less than 26 mpg is supported by the data.

02.

(a) A pharmaceutical company claims that their existing medication for reducing chronic migraines is effective in 55% of cases. To evaluate a new medication, a clinical trial is conducted with 150 patients who suffer from chronic migraines. The results show that 90 patients experienced a significant reduction in migraine frequency after taking the new medication.

(i) Find a point estimate of population proportion.

(ii) Construct a 95% confidence interval of population proportion.

(iii) Is there sufficient evidence to conclude that the new medication is more effective than the existing one? Use a significance level of $\alpha = 0.05$.

(b) A research student has collected GPAs from 12 undergraduates using a simple random sampling technique. The GPAs are given in Table 2,

Table 2: GPAs of students

3.15	3.46	3.21	3.31	3.78	3.60	2.98	3.06	3.33	3.02	3.86	3.24

Test, at the 0.05 level of significance, whether the sequence of GPAs is random.

03.

(a) Compare and contrast the Mann-Whitney U test and the Wilcoxon Singed Ranks test.

(b) An agricultural company is evaluating the effectiveness of two different fertilizers in enhancing the growth and yield of tomato plants. One fertilizer is less expensive, while the other is more costly. The management wishes to determine whether there is a significant difference in the yield of tomatoes when using these two fertilizers, as there is a preference among the staff to purchase the cheaper option. To conduct this study, 20 tomato plants of the same variety were selected, and all other growth conditions were controlled. Ten of the plants were treated with the cheaper fertilizer, and the remaining ten were treated with the more expensive fertilizer. The yield data for each group is provided in Table 3:

Table 3: The yield data (in kilograms of tomatoes produced per plant)

Yield of Cheaper Fertilizer in Kg (Group 1)	Yield of Expensive Fertilizer in Kg (Group 2)
2.8	3.5
3.1	3.8
3.4	3.6
3.0	3.7
2.7	3.4
2.9	3.0
3.2	3.3
2.6	3.2
3.2	2.9
3.0	2.8

Using an appropriate test, clarify whether the two fertilizers are significantly different at 5% level of significance.

(c) A health researcher is studying the effect of a new diet plan on cholesterol levels. The researcher measures the cholesterol levels of 10 participants before and after they follow the new diet plan for 8 weeks. The cholesterol levels (in mg/dL) for each participant are recorded before and after the diet intervention. The dataset shown in Table 4,

Table 4: The cholesterol levels of participants

Participant	Before Diet (mg/dL)	After Diet (mg/dL)
1	210	200
2	220	215
3	205	210
4	225	210
5	210	200
6	235	236
7	240	235
8	230	220
9	220	223
10	250	245

Using an appropriate test, clarify whether there is a significant difference in the participants' cholesterol levels before and after following the new diet plan at 5% level of significance.

04.

(a) A company is testing the durability of three different types of smartphone screen protectors (Protector A, Protector B, and Protector C). To evaluate their effectiveness, the company conducts a drop test from a height of 1 meter on 15 smartphones, with 5 smartphones using each type of screen protector. The number of drops each screen can withstand before cracking is recorded as in Table 5,

Table 5: Durability of phones (the number of drops)

Protector A	Protector B	Protector C
10	9	12
12	10	10
11	12	14
13	11	13
10	9	11

Test, at the 0.05 level of significance, whether the medians of the durability (in number of drops) across the three types of screen protectors are different.

(b) A tech company is interested in understanding the relationship between the stress levels of its employees and their productivity. The company measures stress levels using a psychological scale (where higher scores indicate higher stress) and productivity as the number of tasks completed in a week. The data are shown in Table 6,

Table 6: Stress level data and weekly productivity data

Employee	Stress Level (Scale 1-100)	Weekly Productivity (Number of Tasks Completed)
1	75	15
2	90	22
3	80	12
4	45	30
5	60	8
6	55	25
7	50	20
8	70	18
9	65	28
10	85	10

(i) Using Spearman's rank correlation, analyze whether there is a statistically significant correlation between stress levels and weekly productivity among employees.

(ii) Interpret the correlation between stress levels and weekly productivity among employees.

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Appendix

Mann-Whitney U Test

$$U_1 = n_1 n_2 + \frac{n_1(n_1 + 1)}{2} - R_1 \qquad U_2 = n_1 n_2 + \frac{n_2(n_2 + 1)}{2} - R_2$$

R_1 = Sum of the ranks for group 1, R_2 = Sum of the ranks for group 2, n_1 = Sample Size of 1st sample, n_2 = Sample size of 2nd sample.

Wald-Wolfowitz two sample runs test

For even number of runs

$$P(R = 2k) = \frac{2\binom{n_1 - 1}{k - 1}\binom{n_2 - 1}{k - 1}}{\binom{n_1 + n_2}{n_1}}$$

For odd number of runs

$$P(R = 2k + 1) = \frac{\binom{n_1 - 1}{k}\binom{n_2 - 1}{k - 1} + \binom{n_2 - 1}{k}\binom{n_1 - 1}{k - 1}}{\binom{n_1 + n_2}{n_1}}$$

R = Number of Runs, n_1 = Sample Size of 1st sample, n_2 = Sample size of 2nd sample.

Kruskal–Wallis H-test

$$H = \frac{12}{N(N + 1)} \sum_{i=1}^{k} \frac{T_i^2}{N_i} - 3(N + 1)$$

T_i = the rank sum of ith sample, N_i = sample size for i^{th} sample, N = Full sample size

Spearman Rank Correlation

$$\rho = 1 - \frac{6 \sum d_i^2}{n(n^2 - 1)}$$

d_i = the difference between the ranks of corresponding variables. n = sample size

One Sample Sign Test (Large Sample Approximation)

$$Z = \frac{\left(S + \frac{1}{2}\right) - \frac{1}{2} n}{\frac{1}{2} \sqrt{n}}$$

$S = \min(N^-, N^+)$, N^+ = Number of positive differences, N^- = Number of negative differences, n = sample size

Wilcoxon Sign Rank Test (Large Sample Approximation)

$$Z = \frac{W - \frac{n(n + 1)}{4}}{\sqrt{\frac{n(n + 1)(2n + 1)}{24}}}$$

$W = \min(W^-, W^+)$, W^+ = the sum of the ranks with positive signs, W^- = the sum of the ranks with negative signs, n = sample size

Large Sample Confidence Interval of Population Proportion

$$\hat{p} - Z_{1-\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} < p < \hat{p} + Z_{1-\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$\hat{p}$ = proportion of the sample with the characteristic of interest
n = sample size

Cumulative Binomial Distribution Table

n	x	$p = 0.05$	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
2	0	0.9025	0.8100	0.7225	0.6400	0.5625	0.4900	0.4225	0.3600	0.3025	0.2500
	1	0.9975	0.9900	0.9775	0.9600	0.9375	0.9100	0.8775	0.8400	0.7975	0.7500
3	0	0.8574	0.7290	0.6141	0.5120	0.4219	0.3430	0.2746	0.2160	0.1664	0.1250
	1	0.9928	0.9720	0.9393	0.8960	0.8438	0.7840	0.7183	0.6480	0.5748	0.5000
	2	0.9999	0.9990	0.9966	0.9920	0.9844	0.9730	0.9571	0.9360	0.9089	0.8750
4	0	0.8145	0.6561	0.5220	0.4096	0.3164	0.2401	0.1785	0.1296	0.0915	0.0625
	1	0.9860	0.9477	0.8905	0.8192	0.7383	0.6517	0.5630	0.4752	0.3910	0.3125
	2	0.9995	0.9963	0.9880	0.9728	0.9492	0.9163	0.8735	0.8208	0.7585	0.6875
	3	1.0000	0.9999	0.9995	0.9984	0.9961	0.9919	0.9850	0.9744	0.9590	0.9375
5	0	0.7738	0.5905	0.4437	0.3277	0.2373	0.1681	0.1160	0.0778	0.0503	0.0313
	1	0.9774	0.9185	0.8352	0.7373	0.6328	0.5282	0.4284	0.3370	0.2562	0.1875
	2	0.9988	0.9914	0.9734	0.9421	0.8965	0.8369	0.7648	0.6826	0.5931	0.5000
	3	1.0000	0.9995	0.9978	0.9933	0.9844	0.9692	0.9460	0.9130	0.8688	0.8125
	4	1.0000	1.0000	0.9999	0.9997	0.9990	0.9976	0.9947	0.9898	0.9815	0.9688
6	0	0.7351	0.5314	0.3771	0.2621	0.1780	0.1176	0.0754	0.0467	0.0277	0.0156
	1	0.9672	0.8857	0.7765	0.6554	0.5339	0.4202	0.3191	0.2333	0.1636	0.1094
	2	0.9978	0.9842	0.9527	0.9011	0.8306	0.7443	0.6471	0.5443	0.4415	0.3438
	3	0.9999	0.9987	0.9941	0.9830	0.9624	0.9295	0.8826	0.8208	0.7447	0.6563
	4	1.0000	0.9999	0.9996	0.9984	0.9954	0.9891	0.9777	0.9590	0.9308	0.8906
	5	1.0000	1.0000	1.0000	0.9999	0.9998	0.9993	0.9982	0.9959	0.9917	0.9844
7	0	0.6983	0.4783	0.3206	0.2097	0.1335	0.0824	0.0490	0.0280	0.0152	0.0078
	1	0.9556	0.8503	0.7166	0.5767	0.4449	0.3294	0.2338	0.1586	0.1024	0.0625
	2	0.9962	0.9743	0.9262	0.8520	0.7564	0.6471	0.5323	0.4199	0.3164	0.2266
	3	0.9998	0.9973	0.9879	0.9667	0.9294	0.8740	0.8002	0.7102	0.6083	0.5000
	4	1.0000	0.9998	0.9988	0.9953	0.9871	0.9712	0.9444	0.9037	0.8471	0.7734
	5	1.0000	1.0000	0.9999	0.9996	0.9987	0.9962	0.9910	0.9812	0.9643	0.9375
	6	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9994	0.9984	0.9963	0.9922
8	0	0.6634	0.4305	0.2725	0.1678	0.1001	0.0576	0.0319	0.0168	0.0084	0.0039
	1	0.9428	0.8131	0.6572	0.5033	0.3671	0.2553	0.1691	0.1064	0.0632	0.0352
	2	0.9942	0.9619	0.8948	0.7969	0.6785	0.5518	0.4278	0.3154	0.2201	0.1445
	3	0.9996	0.9950	0.9786	0.9437	0.8862	0.8059	0.7064	0.5941	0.4770	0.3633
	4	1.0000	0.9996	0.9971	0.9896	0.9727	0.9420	0.8939	0.8263	0.7396	0.6367
	5	1.0000	1.0000	0.9998	0.9988	0.9958	0.9887	0.9747	0.9502	0.9115	0.8555
	6	1.0000	1.0000	1.0000	0.9999	0.9996	0.9987	0.9964	0.9915	0.9819	0.9648
	7	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9993	0.9983	0.9961
9	0	0.6302	0.3874	0.2316	0.1342	0.0751	0.0404	0.0207	0.0101	0.0046	0.0020
	1	0.9288	0.7748	0.5995	0.4362	0.3003	0.1960	0.1211	0.0705	0.0385	0.0195
	2	0.9916	0.9470	0.8591	0.7382	0.6007	0.4628	0.3373	0.2318	0.1495	0.0898
	3	0.9994	0.9917	0.9661	0.9144	0.8343	0.7297	0.6089	0.4826	0.3614	0.2539
	4	1.0000	0.9991	0.9944	0.9804	0.9511	0.9012	0.8283	0.7334	0.6214	0.5000
	5	1.0000	0.9999	0.9994	0.9969	0.9900	0.9747	0.9464	0.9006	0.8342	0.7461
	6	1.0000	1.0000	1.0000	0.9997	0.9987	0.9957	0.9888	0.9750	0.9502	0.9102
	7	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9986	0.9962	0.9909	0.9805
	8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9992	0.9980

n	Two-Tailed Test		One-Tailed Test	
	$\alpha = .05$	$\alpha = .01$	$\alpha = .05$	$\alpha = .01$
5	--	--	0	--
6	0	--	2	--
7	2	--	3	0
8	3	0	5	1
9	5	1	8	3
10	8	3	10	5
11	10	5	13	7
12	13	7	17	9
13	17	9	21	12
14	21	12	25	15
15	25	15	30	19
16	29	19	35	23
17	34	23	41	27
18	40	27	47	32
19	46	32	53	37
20	52	37	60	43
21	58	42	67	49
22	65	48	75	55
23	73	54	83	62
24	81	61	91	69
25	89	68	100	76
26	98	75	110	84
27	107	83	119	92
28	116	91	130	101
29	126	100	140	110
30	137	109	151	120

Critical Values of r in the runs test

Given in the tables are various critical values of r for values of m and n less than or equal to 20. For the one-sample runs test, any observed value of r which is less than or equal to the smaller value, or is greater than or equal to the larger value in a pair is significant at the $\alpha = .05$ level.

Each cell shows the pair of critical values as *smaller / larger* ("-" indicates no value).

m \ n	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2											2/-	2/-	2/-	2/-	2/-	2/-	2/-	2/-	2/-
3					2/-	2/-	2/-	2/-	2/-	2/-	2/-	2/-	2/-	3/-	3/-	3/-	3/-	3/-	3/-
4				2/9	2/9	2/-	3/-	3/-	3/-	3/-	3/-	3/-	3/-	3/-	4/-	4/-	4/-	4/-	4/-
5			2/9	2/10	3/10	3/11	3/11	3/-	3/-	4/-	4/-	4/-	4/-	4/-	4/-	4/-	5/-	5/-	5/-
6		2/-	2/9	3/10	3/11	3/12	3/12	4/13	4/13	4/13	4/13	5/-	5/-	5/-	5/-	5/-	5/-	6/-	6/-
7		2/-	2/-	3/11	3/12	3/13	4/13	4/14	5/14	5/14	5/14	5/15	5/15	6/15	6/-	6/-	6/-	6/-	6/-
8		2/-	3/-	3/11	3/12	4/13	4/14	5/14	5/15	5/15	6/16	6/16	6/16	6/16	6/17	7/17	7/17	7/17	7/17
9		2/-	3/-	3/-	4/13	4/14	5/14	5/15	5/16	6/16	6/16	6/17	7/17	7/18	7/18	7/18	8/18	8/18	8/18
10		2/-	3/-	3/-	4/13	5/14	5/15	5/16	6/16	6/17	7/17	7/18	7/18	7/18	8/19	8/19	8/19	8/20	9/20
11		2/-	3/-	4/-	4/13	5/14	5/15	6/16	6/17	7/17	7/18	7/19	8/19	8/19	8/20	9/20	9/20	9/21	9/21
12	2/-	2/-	3/-	4/-	4/13	5/14	6/16	6/16	7/17	7/18	7/19	8/19	8/20	8/20	9/21	9/21	9/21	10/22	10/22
13	2/-	2/-	3/-	4/-	5/-	5/15	6/16	6/17	7/18	7/19	8/19	8/20	9/20	9/21	9/21	10/22	10/22	10/23	10/23
14	2/-	2/-	3/-	4/-	5/-	5/15	6/16	7/17	7/18	8/19	8/20	9/20	9/21	9/22	10/22	10/23	10/23	11/23	11/24
15	2/-	3/-	3/-	4/-	5/-	6/15	6/16	7/18	7/18	8/19	8/20	9/21	9/22	10/22	10/23	11/23	11/24	11/24	12/25
16	2/-	3/-	4/-	4/-	5/-	6/-	6/17	7/18	8/19	8/20	9/21	9/21	10/22	10/23	11/23	11/24	11/25	12/25	12/25
17	2/-	3/-	4/-	4/-	5/-	6/-	7/17	7/18	8/19	9/20	9/21	10/22	10/23	11/23	11/24	11/25	12/25	12/26	13/26
18	2/-	3/-	4/-	5/-	5/-	6/-	7/17	8/18	8/19	9/20	9/21	10/22	10/23	11/24	11/25	12/25	12/26	13/26	13/27
19	2/-	3/-	4/-	5/-	6/-	6/-	7/17	8/18	8/20	9/21	10/22	10/23	11/23	11/24	12/25	12/26	13/26	13/27	13/27
20	2/-	3/-	4/-	5/-	6/-	6/-	7/17	8/18	9/20	9/21	10/22	10/23	11/24	12/25	12/25	13/26	13/27	13/27	14/28

* Adapted from Swed, and Eisenhart, C. (1943). Tables for testing randomness of grouping in a sequence of alternatives. *Annals of Mathematical Statistics*, 14, 83–86, with the kind permission of the authors and publisher.

Critical Values of the Mann – Whitney U

n_2	α	n_1																	
		3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
3	.05	--	0	0	1	1	2	2	3	3	4	4	5	5	6	6	7	7	8
	.01	--	0	0	0	0	0	0	0	0	1	1	1	2	2	2	2	3	3
4	.05	--	0	1	2	3	4	4	5	6	7	8	9	10	11	11	12	13	14
	.01	--	--	0	0	0	1	1	2	2	3	3	4	5	5	6	6	7	8
5	.05	0	1	2	3	5	6	7	8	9	11	12	13	14	15	17	18	19	20
	.01	--	--	0	1	1	2	3	4	5	6	7	7	8	9	10	11	12	13
6	.05	1	2	3	5	6	8	10	11	13	14	16	17	19	21	22	24	25	27
	.01	--	0	1	2	3	4	5	6	7	9	10	11	12	13	15	16	17	18
7	.05	1	3	5	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34
	.01	--	0	1	3	4	6	7	9	10	12	13	15	16	18	19	21	22	24
8	.05	2	4	6	8	10	13	15	17	19	22	24	26	29	31	34	36	38	41
	.01	--	1	2	4	6	7	9	11	13	15	17	18	20	22	24	26	28	30
9	.05	2	4	7	10	12	15	17	20	23	26	28	31	34	37	39	42	45	48
	.01	0	1	3	5	7	9	11	13	16	18	20	22	24	27	29	31	33	36
10	.05	3	5	8	11	14	17	20	23	26	29	33	36	39	42	45	48	52	55
	.01	0	2	4	6	9	11	13	16	18	21	24	26	29	31	34	37	39	42
11	.05	3	6	9	13	16	19	23	26	30	33	37	40	44	47	51	55	58	62
	.01	0	2	5	7	10	13	16	18	21	24	27	30	33	36	39	42	45	48
12	.05	4	7	11	14	18	22	26	29	33	37	41	45	49	53	57	61	65	69
	.01	1	3	6	9	12	15	18	21	24	27	31	34	37	41	44	47	51	54
13	.05	4	8	12	16	20	24	28	33	37	41	45	50	54	59	63	67	72	76
	.01	1	3	7	10	13	17	20	24	27	31	34	38	42	45	49	53	56	60
14	.05	5	9	13	17	22	26	31	36	40	45	50	55	59	64	67	74	78	83
	.01	1	4	7	11	15	18	22	26	30	34	38	42	46	50	54	58	63	67
15	.05	5	10	14	19	24	29	34	39	44	49	54	59	64	70	75	80	85	90
	.01	2	5	8	12	16	20	24	29	33	37	42	46	51	55	60	64	69	73
16	.05	6	11	15	21	26	31	37	42	47	53	59	64	70	75	81	86	92	98
	.01	2	5	9	13	18	22	27	31	36	41	45	50	55	60	65	70	74	79
17	.05	6	11	17	22	28	34	39	45	51	57	63	67	75	81	87	93	99	105
	.01	2	6	10	15	19	24	29	34	39	44	49	54	60	65	70	75	81	86
18	.05	7	12	18	24	30	36	42	48	55	61	67	74	80	86	93	99	106	112
	.01	2	6	11	16	21	26	31	37	42	47	53	58	64	70	75	81	87	92
19	.05	7	13	19	25	32	38	45	52	58	65	72	78	85	92	99	106	113	119
	.01	3	7	12	17	22	28	33	39	45	51	56	63	69	74	81	87	93	99
20	.05	8	14	20	27	34	41	48	55	62	69	76	83	90	98	105	112	119	127
	.01	3	8	13	18	24	30	36	42	48	54	60	67	73	79	86	92	99	105

Chi-square Distribution Table

d.f.	.995	.99	.975	.95	.9	.1	.05	.025	.01
1	0.00	0.00	0.00	0.00	0.02	2.71	3.84	5.02	6.63
2	0.01	0.02	0.05	0.10	0.21	4.61	5.99	7.38	9.21
3	0.07	0.11	0.22	0.35	0.58	6.25	7.81	9.35	11.34
4	0.21	0.30	0.48	0.71	1.06	7.78	9.49	11.14	13.28
5	0.41	0.55	0.83	1.15	1.61	9.24	11.07	12.83	15.09
6	0.68	0.87	1.24	1.64	2.20	10.64	12.59	14.45	16.81
7	0.99	1.24	1.69	2.17	2.83	12.02	14.07	16.01	18.48
8	1.34	1.65	2.18	2.73	3.49	13.36	15.51	17.53	20.09
9	1.73	2.09	2.70	3.33	4.17	14.68	16.92	19.02	21.67
10	2.16	2.56	3.25	3.94	4.87	15.99	18.31	20.48	23.21
11	2.60	3.05	3.82	4.57	5.58	17.28	19.68	21.92	24.72
12	3.07	3.57	4.40	5.23	6.30	18.55	21.03	23.34	26.22
13	3.57	4.11	5.01	5.89	7.04	19.81	22.36	24.74	27.69
14	4.07	4.66	5.63	6.57	7.79	21.06	23.68	26.12	29.14
15	4.60	5.23	6.26	7.26	8.55	22.31	25.00	27.49	30.58
16	5.14	5.81	6.91	7.96	9.31	23.54	26.30	28.85	32.00
17	5.70	6.41	7.56	8.67	10.09	24.77	27.59	30.19	33.41
18	6.26	7.01	8.23	9.39	10.86	25.99	28.87	31.53	34.81
19	6.84	7.63	8.91	10.12	11.65	27.20	30.14	32.85	36.19
20	7.43	8.26	9.59	10.85	12.44	28.41	31.41	34.17	37.57
22	8.64	9.54	10.98	12.34	14.04	30.81	33.92	36.78	40.29
24	9.89	10.86	12.40	13.85	15.66	33.20	36.42	39.36	42.98
26	11.16	12.20	13.84	15.38	17.29	35.56	38.89	41.92	45.64
28	12.46	13.56	15.31	16.93	18.94	37.92	41.34	44.46	48.28
30	13.79	14.95	16.79	18.49	20.60	40.26	43.77	46.98	50.89
32	15.13	16.36	18.29	20.07	22.27	42.58	46.19	49.48	53.49
34	16.50	17.79	19.81	21.66	23.95	44.90	48.60	51.97	56.06
38	19.29	20.69	22.88	24.88	27.34	49.51	53.38	56.90	61.16
42	22.14	23.65	26.00	28.14	30.77	54.09	58.12	61.78	66.21
46	25.04	26.66	29.16	31.44	34.22	58.64	62.83	66.62	71.20
50	27.99	29.71	32.36	34.76	37.69	63.17	67.50	71.42	76.15
55	31.73	33.57	36.40	38.96	42.06	68.80	73.31	77.38	82.29
60	35.53	37.48	40.48	43.19	46.46	74.40	79.08	83.30	88.38
65	39.38	41.44	44.60	47.45	50.88	79.97	84.82	89.18	94.42
70	43.28	45.44	48.76	51.74	55.33	85.53	90.53	95.02	100.43
75	47.21	49.48	52.94	56.05	59.79	91.06	96.22	100.84	106.39
80	51.17	53.54	57.15	60.39	64.28	96.58	101.88	106.63	112.33
85	55.17	57.63	61.39	64.75	68.78	102.08	107.52	112.39	118.24
90	59.20	61.75	65.65	69.13	73.29	107.57	113.15	118.14	124.12
95	63.25	65.90	69.92	73.52	77.82	113.04	118.75	123.86	129.97
100	67.33	70.06	74.22	77.93	82.36	118.50	124.34	129.56	135.81