UNIVERSITY OF KELANIYA – SRI LANKA
CENTRE FOR DISTANCE AND CONTINUING EDUCATION

Bachelor of Science (General) Degree (External) Examination – September 2024

Third Year Semester I(2019 Intake)

STATISTICS

STAT 36513 – Time Series Analysis

No. of questions: Four (04) **No. of pages: Six (06)** **Duration: Two and Half (2 1/2) hrs.**

Answer all questions

01.

a. Define what is a Time Series.

b. What is meant by a stationary time series?

c. Explain one statistical test that can be used to assess the stationarity of a time series.

d. State four main components of a time series.

e. Identify component/s of below two-time series plots with suitable justifications.

Monthly Average CO_2 Concentration

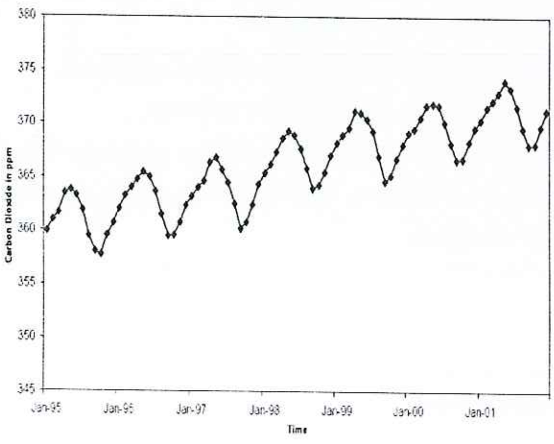


Figure 1

Daily Apple Stock Price

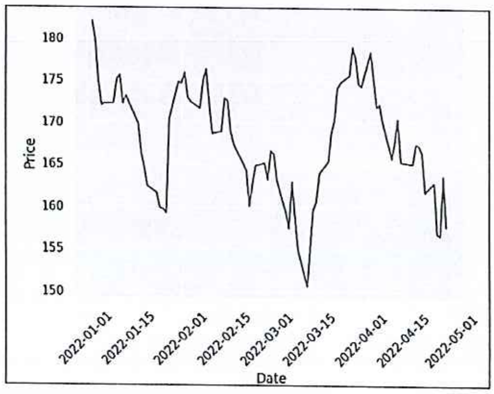


Figure 2

02.

a. Briefly explain the following trend elimination techniques.

   (i). Regression

   (ii). Smoothing with Moving Average filters.

b. Grand National Hotel has been operating in Kenya for the past 15 years. The quarterly profits of the hotel during the last four years were as follows.

Profits (Kenyan Shilling "Million")

Year	Quarter			
	1	2	3	4
2020	33	36	35	38
2021	42	40	42	47
2022	54	53	54	62
2023	70	67	70	77

(i). Apply the centered four quarter moving average filters to eliminate the trend component in the above data set.

(ii). Apply the exponential smoothing technique with smoothing constant $\alpha = 0.02$ to the above data to eliminate the trend component.

(iii). Is the exponential smoothing technique applied in part (ii) more suitable to eliminate the trend component in the data set than the technique justified in part (i)? Justify your answer.

(iv). Calculate the quarterly seasonal components for the given data.

03.

a. Briefly explain three major assumptions of time series models. For each technique mention one statistical test to assess each assumption.

b. Define an autoregressive moving average process of order (p,q) (ARMA (p,q) process).

c. Let the time series $\{Y_t\}$ be defined by $Y_t = 0.7y_{t-1} - 0.1y_{t-2} + \varepsilon_t$, where $\{\varepsilon_t\}$ is a white noise with mean zero and variance σ^2.

(i). Identify the time series model given by the above equation.

(ii). Is the above process stationary? Justify your answer.

(iii). Find the mean and autocovariance function of the process MA (1).

$X_t = \varepsilon_t - \theta\varepsilon_{t-1}$; where ε_t is white noise with mean zero and variance σ^2, and θ is a constant.

04.

a. Interpret the following time series plots.

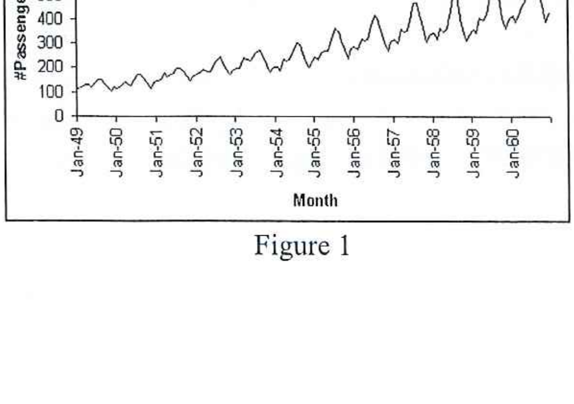


Figure 1

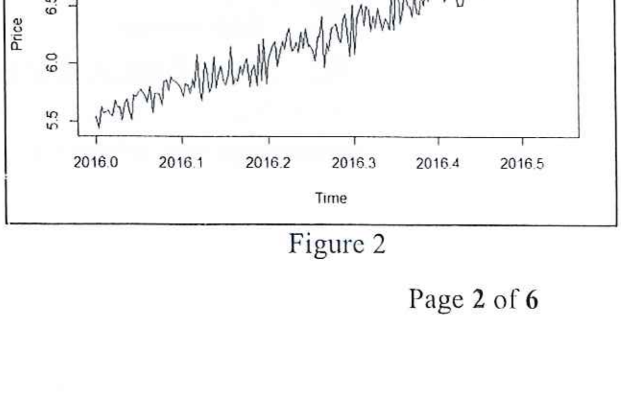


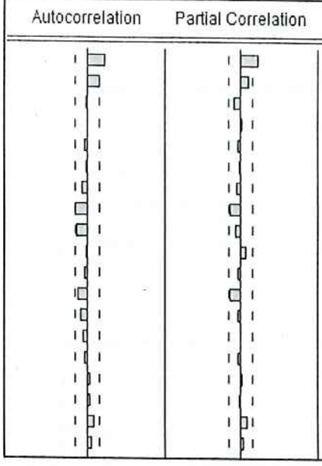
Figure 2

b. ACF and PACF plots of different time series are given below. Write down all candidate time series models for each series using given plots.

(i). Correlogram of the first differenced series of GNP

Correlogram of D(GNP)

Date: 05/29/19 Time: 10:27
Sample: 1947Q1 1981Q2
Included observations: 137

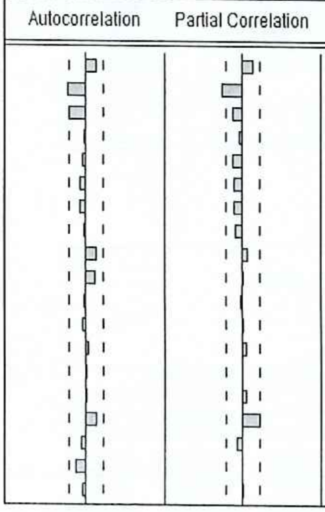


	AC	PAC	Q-Stat	Prob
1	0.244	0.244	8.3285	0.004
2	0.165	0.112	12.177	0.002
3	-0.022	-0.092	12.246	0.007
4	0.008	0.014	12.254	0.016
5	-0.037	-0.025	12.449	0.029
6	-0.020	-0.013	12.507	0.052
7	-0.075	-0.063	13.338	0.064
8	-0.160	-0.141	17.139	0.029
9	-0.143	-0.066	20.165	0.017
10	-0.008	0.079	20.175	0.028
11	-0.027	-0.031	20.282	0.042
12	-0.116	-0.146	22.327	0.034
13	-0.088	-0.036	23.521	0.036
14	-0.058	-0.005	24.044	0.045
15	-0.034	-0.031	24.223	0.061
16	0.038	0.023	24.451	0.080
17	0.029	-0.017	24.583	0.104
18	0.096	0.087	26.065	0.098
19	0.060	0.028	26.654	0.113

(ii). Correlogram of first differenced series of Lake Water Quality

Correlogram of D(LAKE)

Date: 06/18/19 Time: 11:53
Sample: 1 78
Included observations: 77



	AC	PAC	Q-Stat	Prob
1	0.146	0.146	1.7175	0.190
2	-0.237	-0.264	6.2549	0.044
3	-0.209	-0.139	9.8392	0.020
4	-0.022	-0.030	9.8800	0.042
5	-0.032	-0.120	9.9671	0.076
6	-0.076	-0.110	10.463	0.106
7	-0.078	-0.113	10.991	0.139
8	-0.017	-0.082	11.017	0.201
9	0.140	0.073	12.776	0.173
10	0.116	0.024	13.991	0.173
11	-0.017	-0.023	14.018	0.232
12	-0.047	0.013	14.227	0.286
13	0.025	0.042	14.286	0.354
14	0.013	0.003	14.302	0.427
15	0.010	0.055	14.313	0.502
16	0.154	0.224	16.668	0.407
17	-0.059	-0.074	17.025	0.453
18	-0.122	-0.005	18.551	0.420
19	-0.035	0.021	18.683	0.477

c. A statistician has identified the following time series models for some of the company's datasets. Review the provided outputs and write down the model equation for each, explaining your reasoning based on the given information.

(i). MA (4) model

Dependent Variable: D(RAINFALL_DATA)
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 06/10/19 Time: 16:56
Sample: 2 80
Included observations: 79
Convergence achieved after 49 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.028077	0.030124	-0.932061	0.3544
MA(1)	-1.157072	0.176845	-6.542868	0.0000
MA(2)	0.033264	0.172792	0.192509	0.8479
MA(3)	-0.079231	0.178314	-0.444331	0.6581
MA(4)	0.409609	0.181369	2.258429	0.0269
SIGMASQ	1.608199	0.421359	3.816699	0.0003

R-squared	0.632051	Mean dependent var	-0.022258
Adjusted R-squared	0.606849	S.D. dependent var	2.103984
S.E. of regression	1.319234	Akaike info criterion	3.541674
Sum squared resid	127.0477	Schwarz criterion	3.721632
Log likelihood	-133.8961	Hannan-Quinn criter.	3.613771
F-statistic	25.07944	Durbin-Watson stat	2.103949
Prob(F-statistic)	0.000000		

Inverted MA Roots	.94-.30i	.94+.30i	-.36+.54i	-.36-.54i

(ii). AR (2) Model

Dependent Variable: D(LAKE)
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 06/13/19 Time: 09:14
Sample: 2 78
Included observations: 77
Convergence achieved after 17 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.000234	0.073727	0.003178	0.9975
AR(1)	0.203212	0.108669	1.870013	0.0655
AR(2)	-0.308431	0.097079	-3.177114	0.0022
SIGMASQ	0.472973	0.079262	5.967240	0.0000

R-squared	0.105381	Mean dependent var	0.006104
Adjusted R-squared	0.068616	S.D. dependent var	0.731876
S.E. of regression	0.706321	Akaike info criterion	2.195969
Sum squared resid	36.41889	Schwarz criterion	2.317725
Log likelihood	-80.54480	Hannan-Quinn criter.	2.244670
F-statistic	2.866330	Durbin-Watson stat	2.053251
Prob(F-statistic)	0.042359		

Inverted AR Roots	.10+.55i	.10-.55i

(iii). ARMA (2,2) Model

Dependent Variable: EXPORTS
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 08/28/18 Time: 21:21
Sample: 1995Q1 2017Q1
Included observations: 89
Convergence not achieved after 500 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	286468.1	119759.0	2.392037	0.0190
AR(1)	1.542940	0.067046	23.01299	0.0000
AR(2)	-0.546842	0.063580	-8.600848	0.0000
MA(1)	0.023906	0.084534	0.282801	0.7780
MA(2)	0.996916	0.745112	1.337940	0.1846
SIGMASQ	15558450	11415437	1.362931	0.1766

R-squared	0.998120	Mean dependent var	297871.2
Adjusted R-squared	0.998006	S.D. dependent var	91477.45
S.E. of regression	4084.502	Akaike info criterion	19.70451
Sum squared resid	1.38E+09	Schwarz criterion	19.87228
Log likelihood	-870.8505	Hannan-Quinn criter.	19.77213
F-statistic	8811.405	Durbin-Watson stat	1.710003
Prob(F-statistic)	0.000000		

Inverted AR Roots	.99	.55
Inverted MA Roots	-.01-1.00i	-.01+1.00i

d. Following two outputs are related to two fitted models for a time series. Discuss the adequacy of the fitted models and recommend the most suitable model with justifications.

Model 1

Dependent Variable: X
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 08/29/24 Time: 12:50
Sample: 1 80
Included observations: 80
Convergence achieved after 4 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.301919	0.284496	-1.061243	0.2919
AR(1)	0.479206	0.099113	4.834929	0.0000
SIGMASQ	1.418132	0.262289	5.406756	0.0000

R-squared	0.234230	Mean dependent var	-0.290840
Adjusted R-squared	0.214340	S.D. dependent var	1.369433
S.E. of regression	1.213830	Akaike info criterion	3.265479
Sum squared resid	113.4505	Schwarz criterion	3.354805
Log likelihood	-127.6191	Hannan-Quinn criter.	3.301292
F-statistic	11.77622	Durbin-Watson stat	1.530790
Prob(F-statistic)	0.000034		

Inverted AR Roots .48

Model 2

Dependent Variable: X
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 08/29/24 Time: 12:51
Sample: 1 80
Included observations: 80
Convergence achieved after 28 iterations
Coefficient covariance computed using outer product of gradients

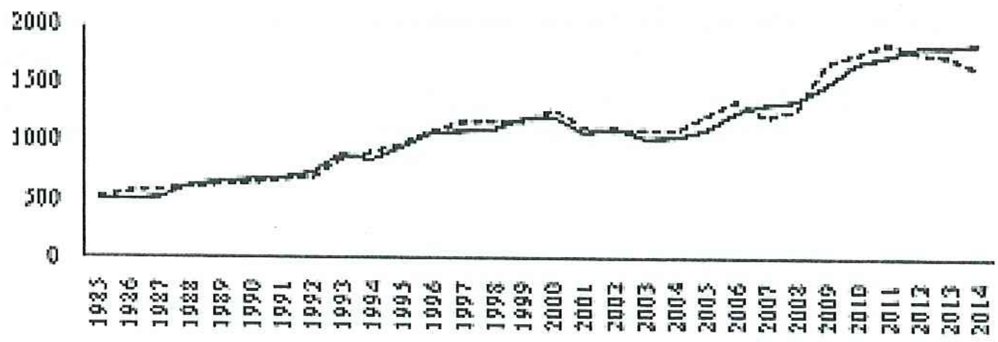
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.283518	0.211109	-1.342996	0.1832
MA(1)	0.751945	0.077668	9.681491	0.0000
SIGMASQ	1.083381	0.217372	4.984001	0.0000

R-squared	0.414991	Mean dependent var	-0.290840
Adjusted R-squared	0.399796	S.D. dependent var	1.369433
S.E. of regression	1.060938	Akaike info criterion	3.003381
Sum squared resid	86.67044	Schwarz criterion	3.092707
Log likelihood	-117.1352	Hannan-Quinn criter.	3.039194
F-statistic	27.31096	Durbin-Watson stat	1.817383
Prob(F-statistic)	0.000000		

Inverted MA Roots -.75

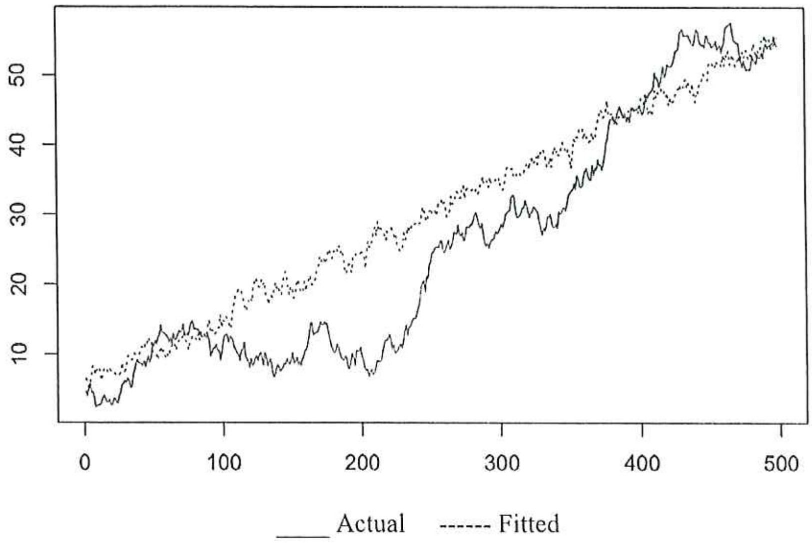
e. A researcher has drawn following graphs to visualize actual verses fitted values for the test set of three-time series. Interpret each graph separately.

Graph 1: Yearly Mango Production Data Set

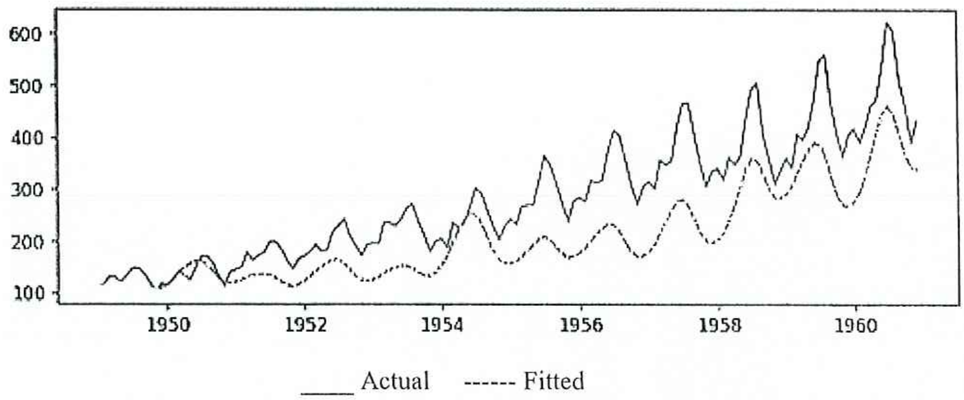


------- Actual ______ Fitted

Graph 2: Daily Sales Data Set



Graph 3: Air Passengers Data Set



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