University of Kelaniya – Sri Lanka
Centre for Distance & Continuing Education
Bachelor of Science (General) External
Second year second semester examination – 2023 (2026 July)
(New Syllabus)
Faculty of Science
Statistics
STAT 27533 – Inferential Statistics

No. of Questions: Five (05) | No. of Pages: Three (03) | Time: **Two & half (2 1/2) Hours.**

Answer **Four (04)** questions only.

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1. *(a)* Briefly describe the following methods used in point estimation:

   (i) Method of Moments,

   (ii) Method of Maximum Likelihood.

   (b) Let $X_1, X_2, \cdots, X_n$ be a random sample of size n taken from a distribution with probability density function

   $$f(x;\ \theta) = \theta^{-2} x e^{-x/\theta}; x > 0, \theta > 0.$$

   (i) Find the Maximum likelihood estimator, $\hat{\theta}_{MLE}$ of θ.

   (ii) Find the Method of Moment estimator, $\hat{\theta}_{MoM}$ of θ.

   (iii) Calculate the estimates when $x_1 = 0.25, x_2 = 0.75,\ x_3 = 1.50,\ x_4 = 2.5\ , x_5 = 2.0$

   [*Hint:* Recall the probability density function of an exponential random variable,

   $$f(x;\theta) = \frac{1}{\theta} e^{-\frac{x}{\theta}};\ x > 0, \theta > 0.$$

   Note that, the moment of this distribution is given by $E[X^k] = \int_0^\infty x^k \frac{1}{\theta} e^{-\frac{x}{\theta}} dx = k!\ \theta^k$.]

2. *(a)* Define the following terms used in estimation theory:

   (i) An Unbiased Estimator,

   (ii) A Sufficient statistic.

   (b) State the Factorization theorem.

(c) Let $X_1, X_2, \cdots, X_n$ be a random sample from the distribution with probability mass function

$$f(x;p) = p^x(1-p)^{1-x}; x = 0, 1.$$

(i) Using Factorization theorem, show that $\sum_{i=1}^{n} X_i$ is a sufficient statistic for p.

(ii) Show that $X_1(1-X_2)$ is an unbiased estimator for $T = p(1-p)$.

(iii) Use Rao-Blackwell theorem to find a better estimator than T for estimating $p(1-p)$.

3. Let X be a single observation from the distribution with probability mass function

$$f(x;p) = \binom{m}{x} p^x(1-p)^{m-x} \; where \; x = 0,1,\cdots,m.$$

Note that the range of X depends on m, but not on the unknown parameter p. Also, the sample size is $n = 1$.

(i) Find the Cramer -Rao lower bound for the unbiased estimator for the parameter p.

(ii) Show that $T_1 = \frac{X}{m}$ is an unbiased estimator for the parameter p. Hence show that it is the most efficient unbiased estimator.

(iii) Let the estimator,

$$T_2 = \frac{X+1}{m+2}.$$

Show that T_2 is not an unbiased estimator for the parameter p.

(iv) Show that the mean-squared error of the estimator T_2, is

$$MSE_{T_2}(p) = \frac{1}{(m+2)^2}[1 + (m-4)p + (4-m)p^2].$$

4.

(a) Define 'a Pivotal Quantity" used in estimation theory.

(b) If the population standard deviation is unknown, construct a $100(1-\alpha)\%$ confidence interval for the population mean, μ. State clearly the assumptions.

(c) A quality control engineer wants to estimate the average lifetime of a new type of battery. A random sample of **16 batteries** is tested, producing the following results:

Sample size: $n = 16$
Sample mean: $\bar{x} = 120$ hours
Sample standard deviation: $s = 8$ hours

Construct a **95% confidence interval** for the true mean battery lifetime.

(d) The service time in queues should not have a large variance; otherwise, the queue tends to build up. A bank regularly checks the service times of its tellers to determine their variance. A random sample of 22 service times (in minutes) gives $S^2 = 8$. Find a 95% confidence interval for the overall variance of service time at the bank.

5.

(a) Briefly describe, Type I error and Type II error as used in hypothesis testing.

(b) (i) Let $X_1, X_2, \cdots, X_n$ be a random sample from the population having the distribution $N(\mu_X, \sigma^2)$ and $Y_1, Y_2, \cdots, Y_m$ be a random sample from the population having the distribution $N(\mu_Y, \sigma^2)$. Assume that the two sample sizes are not too large. Write the test statistics to test the null hypothesis $H_0: \mu_X - \mu_Y = 0$ against the alternative hypothesis $H_0: \mu_X - \mu_Y \neq 0$.

(ii) A botanist is interested in comparing the growth response of dwarf pea stems against two different levels of the hormone indoleacetic acid (IAA). Using 16-day-old pea plants, the botanist obtains 5-mm sections and floats these sections on solutions with different hormone concentrations to observe the effect of the hormone on the growth of the pea stem. Let X and Y denote, respectively, the independent growths that can be attributed to the hormone during the first 26 hours after sectioning for $(0.5)\,(10)^{-4}$ and 10^{-4} levels of concentration of IAA.

The measured growths of pea stem segments, in millimeters, for $n = 11$ observations of X:

0.8 1.8 1.0 0.1 0.9 1.7 1.0 1.4 0.9 1.2 0.5.

The measured growths of pea stem segments, in millimeters, for m = 13 observations of Y:

1.0 0.8 1.6 2.6 1.3 1.1 2.4 1.8 2.5 1.4 1.9
2.0 1.2.

Determine the null and alternative hypotheses and test the hypotheses at 5% significance level using the critical value approach and interpret the results.

(c) The number of typing errors in a newspaper can be modeled by a Poisson distribution with rate parameter λ. Each of the 31 students in a journalism course is allocated at random a past edition of the newspaper and asked to find the number of typing errors it contains. It has been hypothesized by the instructor of the course that there is an average of 5 typing errors in each edition of the newspaper. The editor of the newspaper has objected, claiming that the mean number of errors is less than 5 per edition.

Using the Neyman-Pearson lemma, find the most powerful test with significance level α to test the hypotheses, $H_0 : \lambda = 5$ vs $H_1 : \lambda = 3$.

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