

University of Kelaniya – Sri Lanka
Centre for Distance & Continuing Education
Bachelor of Science (General) External
Second year First semester examination - January 2026
(New Syllabus)
Faculty of Science

Statistics
STAT 26513/R – Probability Distributions & Applications II

No. of Questions: Five (05)

No. of Pages: Four (04)

Time: Two & half (2 1/2) Hours.

Answer **Four (04)** questions only.

1. A privately owned business operates both a drive-in facility and a walk-in facility. On a randomly selected day, let X and Y , respectively, be the proportions of the time that the drive-in and the walk-in facilities are in use, and suppose that the joint density function of these random variables is given by

$$f(x, y) = \begin{cases} \frac{2}{5}(2x + 3y), & 0 \leq x \leq 1, 0 \leq y \leq 1, \\ 0, & \text{otherwise} \end{cases}$$

- (a) Verify $f(x, y)$ is a joint density function of the continuous random variables X and Y .
[10 Marks]

- (b) Find the probability $P[(x, y) \in A]$, where $A = \{(x, y) | 0 < x < \frac{1}{2}, \frac{1}{4} < y < \frac{1}{2}\}$.

[15 Marks]

2. A delivery truck travels from point A to point B and back using the same route each day. There are four traffic lights on the route. Let X_1 denote the number of red lights the truck encounters going from A to B and X_2 denote the number encountered on the return trip. Data collected over a long period suggest that the joint probability distribution for (X_1, X_2) can be expressed by the following table:

	X_2				
X_1	0	1	2	3	4
0	0.01	0.01	0.03	0.07	0.01
1	0.03	0.05	0.08	0.03	0.02
2	0.03	0.11	0.15	0.01	0.01
3	0.02	0.07	0.1	0.03	0.01
4	0.01	0.06	0.03	0.01	0.01

- Find the marginal densities of X_1 and X_2 .
- Find the conditional probability distribution of X_1 given $X_2 = 3$.
- Calculate $E(X_1)$ and $E(X_2)$.
- Find $E(X_1|X_2 = 3)$.
- Find the standard deviation of X_1 . [5×5 Marks]

3. (a) Let a, b and c be real positive numbers such that $a + b + c + d = 1$.

Prove that, $\frac{1}{2} \leq \frac{a^2}{a+b} + \frac{b^2}{b+c} + \frac{c^2}{c+d} + \frac{d^2}{d+a}$. [10 Marks]

- (b) Let X and Y be two independent random variables with respective moment-generating functions

$$m_X(t) = \frac{1}{1-5t}, \text{ if } t < \frac{1}{5}, \text{ and } m_Y(t) = \frac{1}{(1-5t)^2}, \text{ if } t < \frac{1}{5}.$$

Find $E[(X + Y)^2]$. [15 Marks]

4. (a) The length of bolts produced by a machine has an unknown distribution with a mean 3.03 cm and a standard deviation 0.20 cm.

A random sample of 100 bolts is taken from production.

- (i) Estimate the probability that the mean length of this sample is less than 3 cm. [5 Marks]

- (ii) A second random sample is taken from production. The probability that the *mean* of this sample is less than 3cm needs to be less than 1%.

Find the minimum sample size required.

[10 Marks]

- (b) A six-sided dice is changed so that there are three faces marked 1, two faces marked 3 and one face marked 6. The dice is rolled 40 times and the mean of the 40 scores is recorded. Find the approximate distribution for the mean of the scores.

[10 Marks]

5. (a) Suppose that $X_i \sim N(0,4)$ for $i = 1,2,3$. Assuming that, all these random variables are independent, derive the value of k such that

$$P(X_1 < k\sqrt{X_2^2 + X_3^2}) = 0.01.$$

[15 Marks]

- (b) The independent random variables X_1 and X_2 are each normally distributed with a mean 0 and a variance of 4. Derive values for k of the following:

$$P(3X_1 + 4X_2 > 5) = k.$$

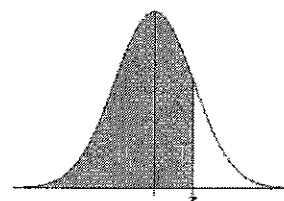
[10 Marks]

Statistical Tables

Critical values of chi-square (right tail)

Degrees of freedom (df)	Significance level (α)							
	.99	.975	.95	.9	.1	.05	.025	.01
1	-----	0.001	0.004	0.016	2.706	3.841	5.024	6.635
2	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210
3	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345
4	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277
5	0.554	0.831	1.145	1.610	9.236	11.070	12.833	15.086
6	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812
7	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475
8	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090
9	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666
10	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209
11	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725
12	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217
13	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688
14	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141
15	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578

Standard Normal Cumulative Probability Table



Cumulative probabilities for POSITIVE z-values are shown in the following table:

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767

t Distribution: Critical Values of t

Degrees of freedom	Two-tailed test: One-tailed test:	Significance level					
		10%	5%	2%	1%	0.2%	0.1%
1		6.314	12.706	31.821	63.657	318.309	636.619
2		2.920	4.303	6.965	9.925	22.327	31.599
3		2.353	3.182	4.541	5.841	10.215	12.924
4		2.132	2.776	3.747	4.604	7.173	8.610
5		2.015	2.571	3.365	4.032	5.893	6.869
6		1.943	2.447	3.143	3.707	5.208	5.959
7		1.894	2.365	2.998	3.499	4.785	5.408
8		1.860	2.306	2.896	3.355	4.501	5.041
9		1.833	2.262	2.821	3.250	4.297	4.781
10		1.812	2.228	2.764	3.169	4.144	4.587

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