University of Kelaniya- Sri Lanka
Centre for Distance & Continuing Education
Bachelor of Science (General) External
Second year First semester examination - 2019
(New Syllabus)
2025 May Faculty of Science

STAT 26513/(R) – Probability Distributions and Applications II

No of Questions: **Five** (05) No. of pages: **Two** (02) Time: **Two & half** ($2\frac{1}{2}$) Hours

Answer **four(04)** questions only.

---

1. A Supermarket has two express lines. Let X and Y denote the number of customers in the first and in the second, respectively, at any given time. During nonrush hours, the joint probability density function of X and Y is summarized by the following table:

		X			
		0	1	2	3
Y	0	0.1	0.2	0	0
	1	0.2	0.25	0.05	0
	2	0	0.05	0.05	0.025
	3	0	0	0.025	0.05

   (a) Find the probability that the total number of customers in both lines are greater than five during nonrush hours.

   (b) Find the marginal distributions of X and Y.

   (c) Compute the average number of the total customers in both lines during nonrush hours.

   (d) Are the random variables X and Y independent? Justify your answer.

2. Given the joint probability density

$$f(x,y) = \begin{cases} \frac{2}{3}(x+2y) & \text{for} \quad 0 < x < 1, 0 < y < 1, \\ 0 & \text{otherwise.} \end{cases}$$

   (a) Find the marginal densities of X and Y.

   (b) Find the conditional density of X given $Y = y$.

   (c) Evaluate $P(X \leq \frac{1}{4}|Y = \frac{1}{2})$

   (d) Find the conditional mean of X given $Y = \frac{1}{2}$.

3. Let X and Y have the joint probability mass function

$$f(x,y) = \frac{x+2y}{18}, \quad x = 1,2 \quad y = 1,2.$$

(a) Find the marginal distributions of X and Y.

(b) Find the means and variances of X and Y.

(c) Compute the covariance of X and Y.

(d) Find and interpret the correlation coefficient of X and Y.

(e) Are X and Y independent? Explain your answer.

4. (a) X, Y and Z are two random variables and a, b, c and d are constant. Using the rules of the expected values show that

i. $Cov(aX+b, cY+d) = acCov(X,Y)$;

ii. $Cor(aX+b, cY+d) = Cor(X,Y)$, when a and c have the same sign.

(b) Ms.Pretty and Mr.Busy have agreed to meet for lunch between noon (0:00 P.M.) and 1:00 P.M. Denote Ms.Pretty's arrival time by X, Mr.Busy's by Y, and suppose X and Y are independent with pdf's

$$f_X(x) = \begin{cases} 3x^2 & if \quad 0 \le x \le 1, \\ 0 & \text{otherwise.} \end{cases}$$

$$f_Y(y) = \begin{cases} 2y & if \quad 0 \le y \le 1, \\ 0 & \text{otherwise.} \end{cases}$$

What is the expected amount of time that the one who arrives first must wait for the other person? [Hint: $\mid X - Y \mid$.]

5. An engineer measures the amount (by weight) of particulate pollution in air samples of a certain volume collected over two smoke-stacks at a coal-operated power plant. One of the stacks is equipped with a cleaning device. Let Y_1 denote the amount of pollutant per sample collected above stack that has no cleaning device and let Y_2 denote the amount of pollutant per sample collected above stack that is equipped with the cleaning device. The joint probability density of y_1 and y_2 is given by

$$f(y_1, y_2) = \begin{cases} 1 & 0 \le y_1 \le 2, 0 \le y_2 \le 1, 2y_2 \le y_1, \\ 0 & \text{elsewhere.} \end{cases}$$

(a) Find $E(Y_1)$ and $E(Y_2)$.

(b) Find $Var(Y_1)$ and $Var(Y_2)$.

(c) The random variable $Y_1 - Y_2$ represents the amount by which the weight of pollutant can be reduced by using the cleaning device. Find $E(Y_1 - Y_2)$.

(d) Are random variables Y_1 and Y_2 independent? Justify your answer.

(e) Find $Var(Y_1 - Y_2)$.