UNIVERSITY OF KELANIYA - SRI LANKA
FACULTY OF SCIENCE

Bachelor of Science (General) Degree Examination,

Examination June, 2025

Academic Year 2014/2015 - Semester II

APPLIED MATHEMATICS

AMAT 3043- Fluid Dynamics

No. of Questions: Seven(07)   No. of Pages: Four (04)   Time: Two Half $(2\frac{1}{2})$ hrs

Answer **Five (05)** Questions Only.

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1. a) In the usual notation, derive the following equation of continuity for a fluid flow

$$\frac{\partial \rho}{\partial t} + \underline{\nabla}\,.(\rho \underline{q}) = 0 \quad .$$

b) Show that for a homogeneous and incompressible fluid above equation reduces to the following form,

$$\underline{\nabla}\,.\underline{q} = 0 \quad .$$

c) Consider the velocity field given by $\mathbf{q} = (1 + At)\boldsymbol{i} + x\boldsymbol{j}$. Find the equation of streamlines at $t = t_0$ passing through the point (x_0, y_0).

d) Show that $\phi = (x - t)(y - t)$ represents the velocity potential of an incompressible two dimensional fluid.

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2. a) Two sources each of strength $\boldsymbol{m}$, are placed at the points $(-a, 0)$ and $(a, 0)$,

and a sink of strength $\boldsymbol{2m}$ is placed at origin. Show that the streamlines are

the curves

$$(x^2 + y^2)^2 = a^2\,(x^2 - y^2 + \lambda xy)$$ where λ is a variable parameter.

b) Let $w = U\left(z + \frac{a^2}{z}\right) + iklog\left(\frac{z}{a}\right)$ be the complex potential of a two dimensional motion of a fluid, where U, a, k are constants. Show that :

i) the velocity at infinity is U in the negative sense of real axis;

ii) the circle $|z| = a$ is a streamline.

3. a) Starting from the Euler equation, prove the Bernoulli's equation in the following form

$$-\frac{\partial\phi}{\partial t} + \frac{1}{2}q^2 + V + \int \frac{dp}{d\rho} = F(t)$$

where $F(t)$ is an arbitrary function of t arising from the integration.

b) Fluid is coming out from a small hole of cross-section σ_1 in a tank. If the minimum cross-section of the stream coming out of the hole is σ_2, then show that $\frac{\sigma_2}{\sigma_1} = \frac{1}{2}$.

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4. a) Prove that the stream function of a fluid flow, ψ is constant along a given stream line.

   b) Show that the flux across any curve joining two points is given by the difference of the values of the stream functions at two points.

   c) Verify that the stream function ψ of two dimensional incompressible irrotational motion satisfy Laplace's equation $\nabla^2\psi = 0$.

5. a) For a two-dimensional flow the velocity function is given by the expression,

$$\phi = x^2 - y^2.$$

   i. Determine velocity components in x and y directions

   ii. Show that the velocity components satisfy the conditions of flow continuity and irrotationality

   iii. Determine the stream function.

   b) Suppose that the complex potential of two dimensional fluid flow is given by

   $w = z^2$. Show that:

   (i) the streamlines are the rectangular hyperbolae $xy = const.$

   (ii) the equipotentials are the rectangular hyperbolae $x^2 - y^2 = const.$

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6. a) Obtain Euler's equation of motion in Cartesian form.

   b) A sphere of radius a is alone in an unbounded liquid which is rest at a great distance from the sphere and is subject to no external forces. The sphere is forced to vibrate radially keeping its spherical shape, the radius r at any time being given by $r = a + b\cos(nt)$. Show that if Π is the pressure in the liquid at a great distance from the sphere, the least pressure at the surface of the sphere during the motion is $\Pi - n^2\rho b(a+b)$.

7. a) Obtain the following Cauchy-Riemann equations for a incompressible irrotational fluid motion.

$$\frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y}, \quad \frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x}.$$

   b) A two-dimensional flow field is given by $\psi = xy$.

      i. Show that the flow is irrotational.
      ii. Find the velocity potential.
      iii. Verify that ϕ and ψ satisfy the Laplace's equation.
      iv. Find the streamlines and potential lines.

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