University of Kelaniya-Sri Lanka
Centre for Distance & Continuing Education
Bachelor of Science (General) External
Second Year First Semester Examination-2023 (2026-January)
(New Syllabus)
Faculty of Science

Applied Mathematics
AMAT 26562- Mechanics II

No. of Questions: Five (05) **No. of Pages: Three(03)** **Time: Two(02) hrs**

Answer Four (04) questions only.

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01. (a) State and prove the theorem of perpendicular axis for the moment of inertia of a rigid body.

[25 Marks]

Suppose a rectangular plate of mass M has sides $2a$ and $2b$.
Find the moment of inertia about
(i) each side,
(ii) an axis perpendicular to the plate and passing through a vertex,
of the rectangular plate.

[20 Marks]

(b) In the usual notation, show that

$$\frac{dH}{dt} = \sum_{i=1}^{N} \underline{r_i} \times \underline{F_i}\,.$$

[20 Marks]

(c) A uniform rod AB of mass M and length $2a$ is attached to a smooth hinge at point A, while a particle of mass m is attached to the other end B. Initially the rod is kept in a vertical position such that the end B is above A. The system is allowed to fall freely keeping the point A fixed in the space. Using part (b) above, show that

$$\dot{\theta}^2 = \frac{3(M+2m)(1-\cos\theta)g}{2a(M+3m)}$$

where θ is the angle between the rod and the upward vertical.

[35 Marks]

Continued...

02. (a) State clearly what is meant by the statement that two systems S_1 and S_2 are equimomental.

[10 Marks]

(b) State the necessary and sufficient conditions for two systems of particles to be equimomental.

[10 Marks]

(c) Show that a uniform rod of mass m, and length $2a$, is equimomental with a system of three particles of masses $\frac{m}{6}$ each at the end points and $\frac{2m}{3}$ at the mid point.

[30 Marks]

(d) (i) Show that $2\pi\sqrt{\frac{k^2}{gh}}$ is the time of a complete small oscillation of a compound pendulum, where k is the radius of gyration of the rigid body about the fixed axis and h is the distance of centre of inertia of the body from the fixed axis. g is the gravity.

[20 Marks]

(ii) Define the simple equivalent pendulum.

[05 Marks]

(iii) Find the length of the simple equivalent pendulum, when a circular disc of radius a oscillates about a fixed horizontal axis which is tangent to it.

[25 Marks]

03. (a) Show that the moment of inertia of a uniform solid cylinder of height h, radius a and mass M about its axis is $\frac{1}{2}Ma^2$.

[35 Marks]

(b) Show that, in the usual notation, the kinetic energy of a rigid body moving parallel to a fixed plane, is given by

$$T = \frac{1}{2}MV_G^2 + \frac{1}{2}I\omega^2.$$

[25 Marks]

(c) A solid uniform cylinder of mass M , and radius a, rolls without sliding along a straight line on a smooth horizontal table with velocity v . Show that its total kinetic energy is $\frac{3}{4}Mv^2$.

[15 Marks]

(d) Find the radius of the solid sphere of mass M which bears the same kinetic energy as in above part (c).
[In the usual notation, moments of inertia of a sphere with radius a is given by $\frac{2}{5}Ma^2$.]

[25 Marks]

Continued...

04. (a) A rigid body consists of three point masses 2 kg, 1kg and 4 kg, connected by massless rods. These masses are located at coordinates (1,-1,1), (2,0,2) and (-1,1,0) in meters, respectively.

(i) Find the moments of inertia and products of inertia of the body about X, Y and Z axes.

[30 Marks]

(ii) Compute the inertia matrix of the rigid body.

[05 Marks]

(iii) If the system is rotating with an angular velocity $\underline{\omega}=3\underline{i}-2\underline{j}+4\underline{k}$, find the angular momentum vector of this body.

[10 Marks]

(b) Show that the momental ellipsoid at a point on the edge of a circular base of a thin hemispherical shell is

$$2x^2 + 5(y^2 + z^2) - 3zx = \text{constant}.$$

[You may assume that the moment of inertia of a thin spherical shell about an axis along its diameter is $\frac{2}{3}ma^2$, where m is the mass of the spherical shell and a is the radius.]

[55 Marks]

05. (a) Assuming the principle of virtual work function, in the usual notation,

$$\delta W = \sum_{j=1}^{n} Q_j \cdot \delta q_j = \sum_{i=1}^{N} F_i \cdot \delta r_i ,$$

obtain the Lagrange's equations, $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} = 0,\ j = 1,2,\dots,n,$ for a dynamical system, where T is the kinetic energy of the system.

[35 Marks]

(b) Reduce above system of equations to the following form

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = 0,\ j = 1,2,\dots,n.$$

For a conservative holonomic dynamical system, where L is the Lagrangian.

[15 Marks]

(c) A smooth ring of mass $4m$ slides along a smooth uniform rod AB of length $2a$ and mass M which is free to rotate about A, in a vertical plane. Initially the ring is at B with B is vertically above A and the system is at rest. If the system is gently displaced, show that when the rod makes an angle θ with the upward vertical, the Lagrangian L of the system can be written as

$$L = \frac{2}{3}Ma^2\dot{\theta}^2 + 2m\left[\dot{x}^2 + (2a - x)^2\dot{\theta}^2\right] - Mga\cos\theta - 4mg(2a - x)\cos\theta,$$

where x is the distance of the ring from B.

[50 Marks]

****************THE END****************