| Supervisor's | Signature: |
|--------------|------------|
| | Date: |



UNIVERSITY OF KELANIYA - SRI LANKA

Center for Distance & Continuing Education

Bachelor of Science (General) External

First year second semester examination (Repeat)-2019 (June 2025)

(New Syllabus)

FACULTY OF SCIENCE

Applied Mathematics - AMAT 27582

Scientific Computing Using Appropriate Software II

| Index Number: |
|---------------|
| Course Code: |
| Course Title: |
| Date: |
| Date |

| | MARKS | | | | |
|----------|-------------------|--------------------|--|--|--|
| QUESTION | First Examiner | Second Examiner | | | |
| 1 | | | | | |
| 2 | | | | | |
| 3 | | | | | |
| 4 | | | | | |
| 5 | | | | | |
| TOTAL | | | | | |

| First Examiner's Marks | |
|-------------------------|--|
| Second Examiner's Marks | |
| Total | |
| Average | |

APPLIED MATHEMATICS

AMAT 27582 - Scientific Computing using Appropriate Software II

No. of Questions: Five (05)

No. of Pages: Five (05)

Time Allowed: Two (2) hrs

Instructions to candidates

Programmable Calculators Are Not Allowed

Answer only Four (04) Questions. Question 01 is compulsory and THREE other questions should be attempted from rest of the four questions. You are not allowed to remove any page from this booklet.

1. Answer ALL the following multiple-choice questions by writing the letter corresponding to the correct answer in the table provided below.

Note: Only one letter can be written in each box for each question.

| Question No. | Answer |
|--------------|--------|
| (i) | |
| (ii) | |
| (iii) | |
| (iv) | |
| (v) | |

| Question No. | Answer |
|--------------|--------|
| (vi) | |
| (vii) | |
| (viii) | |
| (ix) | |
| (x) | |

(i) Which syntax is used to print a new line in a fprintf command?

A. \nl

(ii) Which of the following MATLAB functions can be used for polynomial curve fitting of arbitrary degree?

A. fitpoly

D. lsqcurvefit

B. polyval

E. fminsearch

- C. polyfit
- What is the transformed linear form of the saturation growth rate model $y = \frac{ax}{h+x}$? (iii)

 $A. \ln(y) = \ln(a) + bx$

D.
$$ln(y) =$$

B. y = a + bx

D.
$$\ln(y) = ax^{b}$$

E. $\frac{1}{y} = \frac{1}{x} + \frac{1}{a}$

$$C.\frac{1}{y} = \frac{b}{a}\frac{1}{x} + \frac{1}{a}$$

Continued...

- What is the disadvantage of linearizing a nonlinear model for regression?
 - A. Reducing computational cost
 - B. Increasing model complexity
 - C. Biased parameter estimations
 - D. Overfitting the original data
 - E. None of the above
- (v) Which syntax will solve for the following differential equation using the built-in function ode45 with a time interval of 0 to 5?

$$\frac{dy}{dx} = 3y + t, \qquad y(0) = 1.$$

- A. [t, y] = ode45(@(t, y) 3*y + t, [0 5], 0);
- B. [t, y] = ode45(@(t, y) 3*y + t, [0 5], 1);
- C. [t, y] = ode45(@(t, y) 3y + t, [0 5], 1);
- D. [t, y] = ode45(@(t, y) 3*y + t, [0 1], 5);
- E. [t, y] = ode45(@(y, t) 3*y + t, [0 5], 1);
- (vi) What does the following code compute?

$$x = 0:0.1:1;$$

 $y = x.^2;$
 $trapz(x, y);$

- A. The area under the curve $y = x^2$ from 0 to 1
- B. The area under the curve $y = x^2$ from 0 to 0.1
- C. Derivative of the function x^2
- D. Simpson's rule integration of x^2
- E. Tripoidal integration of the curve y = 2x
- (vii) Suppose that A represents the coefficient matrix and b represents the right-hand side vector of a given linear system of equations. In MATLAB, which syntax is used to solve the system Ax = b?
 - A. $x=b\setminus A$
- B. x=b*inv(A) C. $x=A\b$
- D. solve (A,b)
- E. A*x==b
- (viii) Which MATLAB function is used for one-dimensional optimization?
 - A. fminsearch
 - B. fminbnd
 - C. fzero
 - D. lsqcurvefit
 - E. linprog

(ix) What is extrapolation?

- A. Estimating values inside data bounds
- B. Estimating values beyond given data range
- B. Smoothing noisy data
- D. Differentiating discrete data
- F. Finding outliers of a given data set
- (x) What condition is required for guaranteed convergence of iterative methods?
- A. Matrix must be symmetric
- B. Matrix must be diagonally dominant
- C. Matrix must be singular
- D. Matrix must be sparse
- E. None of the above

2.

(a) Write down a MATLAB script file to obtain the plots of the following functions $f_1(x)$ and $f_2(x)$:

$$f_1(x) = \frac{e^x \cos(\pi x)}{(x^2 + 3)}, \quad f_2(x) = e^{-0.1x} \sin(2\pi x) \quad x \in [0, 5].$$

Your code should contain the following specifications:

- i Create 50 equally spaced points in the interval [0, 5] and assign it to the variable x.
- ii Define f_1 and f_2 using anonymous function in MATLAB.
- iii Evaluate the functions at x.
- iv Plot the graph of f_1 as red solid line and f_2 as green dash line in the <u>same figure</u>.
- v Plot should contain a legend and axes should be labeled.

(b)

i Using backward substitution or otherwise, write a MATLAB script file to

solve
$$Ax = b$$
, where $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 4 & 6 \\ 0 & 0 & 2 \end{bmatrix}$, $b = \begin{bmatrix} 1 \\ 7 \\ 4 \end{bmatrix}$ and $x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$.

ii Modify the required lines of your code to get user input for the vector b.

3.

- (a) Write a MATLAB script file to find the minimum of the function $f(x) = 2x^2 + 5x + 4$ near x = -1 using the built-in function fminunc. You need to obtain the function value at the minimum point to be determined.
- (b) A hot object is cooling in a room at temperature $T_{room} = 20^{\circ}C$. According to Newton's Law of Cooling, the rate of temperature change of the object is proportional to the difference between the object's temperature and the room temperature:

$$\frac{dT}{dt} = -k(T - T_{room}),$$
 Continued...

Given that the initial object temperature $T(0) = 100^{0}$ C and the constant k = 0.1. Write a MATLAB script file to

i find the temperature over time from t=0 to t=60 minutes using ODE 45 (), ii plot the temperature distribution vs time.

iii Use fprint command to print in the final time to one decimal place and the corresponding temperature to four decimal places.

4. The composite midpoint rule which approximate integral $I = \int_a^b f(x) dx$ is given by

$$I = h\left(\sum_{j=1}^{n} f(x_j)\right); \quad h = \frac{(b-a)}{n} \quad and \quad x_j = a + \left(j - \frac{1}{2}\right)h$$

where n is the number of subintervals in [a, b] interval.

- (a) Write a MATLAB function file composite_midpoint_rule.m to implement the above method.
- (b) Write a MATLAB script file to evaluate the following integral

$$\int_{1}^{3} \ln(x+1) dx$$

using the function file composite_midpoint_rule.m with n=10.

- (c) Use the MATLAB built-in function quad () to verify the result obtained in part (b).
- 5. The following table contains some data representing a relationship between hours studied and test scores of an experiment conducted for a particular group of students.

| Hours Studied (x) | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|----------------------|----|----|----|----|----|----|----|
| Test scores (y) | 52 | 55 | 61 | 66 | 71 | 75 | 77 |

Suppose that you want to fit a linear regression model of the type $y = a_1x + a_0$ and obtain the goodness of fit R^2 value in the usual notation to the given data set above. The corresponding formula for obtaining slope a_1 , intercept a_0 and R^2 are given below:

$$a_1 = \frac{n\sum x_i y_i - \sum x_i \sum y_i}{n\sum x_i^2 - (\sum x_i)^2} \ , a_0 = \bar{y} - a_1 \bar{x} \ , R^2 = 1 - \left(\frac{\sum (y_i - \bar{y})^2}{\sum (y_i - a_0 - a_1 x_i)^2}\right)$$

Where $\overline{(\cdot)}$ represents the mean value of the data set and n is the length of the data set.

- (a) Write a MATLAB function file Linear_regression.m by taking x, y as inputs to output the values a_1, a_0 and R^2 .
- (b) Write a MATLAB script file to obtain the corresponding linear model for the above data set.
- (c) Explain how you would determine the accuracy of the model based on \mathbb{R}^2 value of the fit.