UNIVERSITY OF KELANIYA - SRI LANKA

Centre for Distance & Continuing Education

Bachelor of Science (General) Degree External

Second Year Second Semester Examination – 2019

(New Syllabus)

2024 February

Faculty of Science

Index Number:.....

Course Code:.....

Course Title:.....

Date:.....

QUESTION	MARKS, %	
	First Examiner	Second Examiner
1		
2		
3		
4		
5		
TOTAL		

First Examiner's Marks, %	
Second Examiner's Marks, %	
Total (200)	
Average	

APPLIED MATHEMATICS

AMAT 27582 – Scientific Computing using Appropriate Software II

No. of Questions: Five (05) **No. of Pages: Ten (10)** **Time Allowed: Two (2) hrs**

Answer Four (04) Questions Only

Instructions to candidates

Programmable Calculators Are Not Allowed

Answer only **Four (04)** Questions.

All questions are to be **answered in the boxes** provided within this booklet.

You are **not allowed** to remove any page from this booklet.

1. (i) Describe the Euler's method for solving the differential equation $\frac{dy}{dx} = f(x, y),\ y(x_0) = y_0$. **[25 marks]**

Continued...

(ii) Complete the following MATLAB function file that implements the Euler's method.

```
function [t,y] = Eulode(dydt,tspan,y0,h)
% [t,y] = Eulode(dydt,tspan,y0,h):

 %   uses Euler's method to integrate an ODE
 % input:
 %   dydt = name of the M-file that evaluates the ODE
 %   tspan = [ti, tf] where ti and tf = initial and
 %           final values of independent variable
 %   y0 = initial value of dependent variable
 %   h = step size
 % output:
 %   t = vector of independent variable
 %   y = vector of solution for dependent variable
  ti = tspan(1);
  tf = tspan(2);
```

```
% generate a vector 't' with step size 'h'
```
[10 marks]

```
% use an 'if' statement to add an additional value of t so that
% range goes from t=ti to tf
```
[15 marks]

```
% preallocate 'y' to improve efficiency
```
[05 marks]

```
% implement the Euler's method
```
[15 marks]

Continued...

(iii) State clearly all the MATLAB commands to solve the differential equation $\frac{dy}{dt} = 4\,e^{0.8\,t} - 0.5\,y,\ y(0) = 0$ in the interval $[0, 10]$ using the above function file.
(Numerical calculations are not needed.)

[30 marks]

2. It is proposed to use ode45 to solve the system of ordinary differential equations

$$\frac{dy_1}{dt} = y_2,\ \frac{dy_2}{dt} = (1 - y_1^2)y_2 - y_1.$$

The ODE45 help is as follows.

```
ODE45  Solve non-stiff differential equations, medium order method.
 [TOUT,YOUT] = ODE45(ODEFUN,TSPAN,Y0) with TSPAN = [T0 TFINAL]
integrates the system of differential equations y' = f(t,y) from
time T0 to TFINAL with initial conditions Y0. ODEFUN is a function
handle. For a scalar T and a vector Y, ODEFUN(T,Y) must return a
column vector corresponding to f(t,y). Each row in the solution
array YOUT corresponds to a time returned in the column vector TOUT.
To obtain solutions at specific times T0,T1,...,TFINAL (all
increasing or all decreasing), use TSPAN = [T0 T1 ... TFINAL].
```

A solution is required for time t running from 0 to 20 seconds. The initial values at $t = 0$ are $y_1(0) = 1$ *and* $y_2(0) = 2$.

(i) Show how you would go about using ode45 to obtain an approximate solution to the equations.

[50 marks]

(ii) What commands are required to obtain a time plot of both y_1 *and* y_2? Axis labels and a legend identifying the curves y_1 *and* y_2 are required.

[20 marks]

Continued...

(iii) What commands are required to produce a state-space or phase plot of y_2 against y_1 without over-writing the first plot? **[30 marks]**

3. A MATLAB function that performs composite Simpson's $1/3$ rule is given below. It is error free but incomplete. Complete this function that calculates the integral.
The formula for the composite Simpson's $1/3$ rule is given by
$I = \frac{h}{3}\left[f(x_1) + 4\sum_{i=2,4,6,\ldots}^{n-1} f(x_i) + 2\sum_{j=3,5,7,\ldots}^{n-2} f(x_j) + f(x_n)\right]$.

```
function I=compoS13(func, a, b, n)
%  I= compoS13(func, a, b, n):
%    Calculate the integral using composite Simpson's 1/3 rule
% Inputs:
%  func=function handle of the function to be integrated
%  a, b= integration limits
%  n=number of points
% Output:
%  I=integral estimate
h=(b-a)/(n-1);
x=linspace(a, b, n);
```

Continued...

```
% Calculate the sum of the odd points, s_odd
```

[40 marks]

```
% Calculate the sum of the even points, s_even
```

[40 marks]

```
% Calculate the integral
```

[20 marks]

4. In curve fitting techniques, the syntax of the built-in functions **polyfit** and **polyval** are as follows:

```
P = POLYFIT(X,Y,N) finds the coefficients of a polynomial P(X) of
degree N that fits the data Y best in a least-squares sense. P is a
row vector of length N+1 containing the polynomial coefficients in
descending powers, P(1)*X^N + P(2)*X^(N-1) +...+ P(N)*X + P(N+1).

Y = POLYVAL(P,X) returns the value of a polynomial P evaluated at

X. P is a vector of length N+1 whose elements are the coefficients

of the polynomial in descending powers.Y = P(1)*X^N + P(2)*X^(N-1)

+ ... + P(N)*X + P(N+1)
```

Continued...

The variables y and z are given as follows:

y	0	2	4	6	8	10	12	14	16	18
z	0.3	0.8	1.3	2.5	5	7	4.6	3.3	1.2	0.9

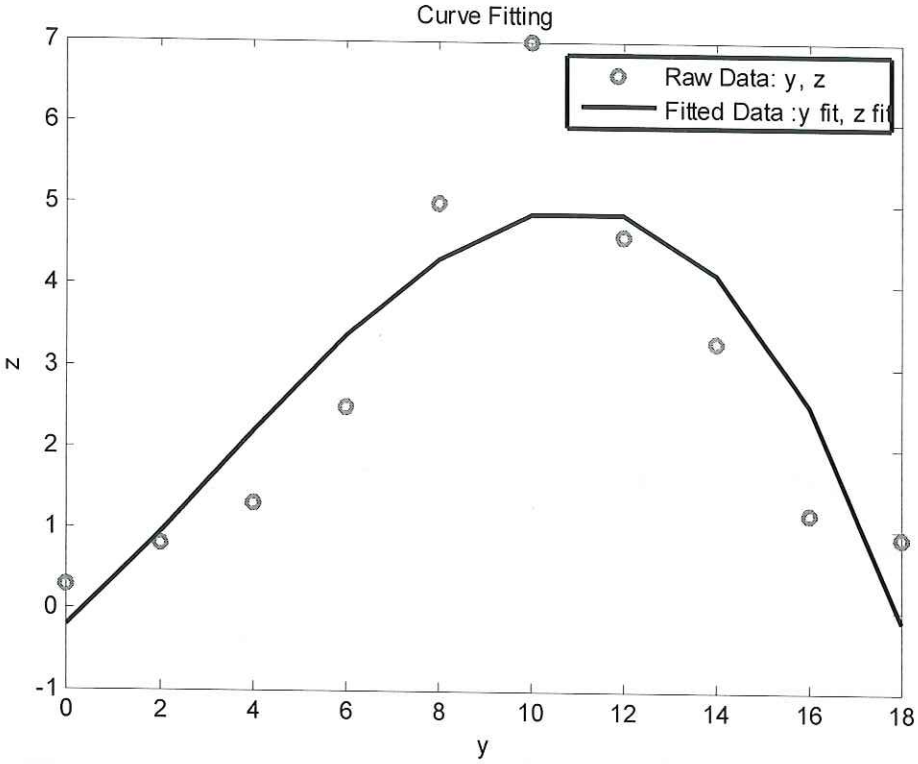
(i) Write a line of MATLAB code to fit a 3rd order polynomial to the arrays y and z using the built-in function **`polyfit`**. **[25 marks]**

(ii) Write a MATLAB code to generate equally spaced data pairs with an interval of 2 along y, using the fitted polynomial in part (a). **[15 marks]**

(iii) Use the first three data points of the table above to find the coefficients of a 2nd order polynomial of the form $z = ay^2 + by + c$. **[15 marks]**

(iv) Write a MATLAB code to generate the figure below using only one plot command. Also, use proper commands to describe the title, axis labels, and legends of the figure. The code should make sure that the limits of the horizontal axis are adjusted to [0, 18], and the limits of the vertical axis are adjusted to [-1, 7]. **[45 marks]**

Continued...



5. (i) The MATLAB function file below calculates an integral using composite trapezoidal rule. But the MATLAB code below has some mistakes. Indicate the line number(s) of the code that is wrong and replace it by the new code. **[50 marks]**

Continued...

```
1  function I = CompTrap(f,a,b,n)
2  % f = function handle of the function to be integrated
3  % a, b = integration limits
4  % n = number of points
5  h = (b-a)/(n-1);
6  x = a:(h-1): b;
7  fx = f(x);
8  s=0;
9  for i = 2 : n-1
10     s = s*fx(i);
11 end
12 I = h/2*(fx(1) + 2*s + fx(j));
```

Line number	New code

(ii) Given that $I = \int_a^b \theta\,(x)dx$.

Calculate the integral I using the data given below. Use Simpson's 1/3 rule to integrate the first 2 segments and use Simpson's 3/8 rule to integrate the last 3 segments. **[20 marks]**

x	0	3	6	9	12	15
$\theta(x)$	0.4	1.2	1.7	1.1	3.1	1.28

Continued...

(iii) Complete the following M-file that uses built-in function `trapz()` to estimate the integral in part (b).

[30 marks]

The MATLAB help for `trapz` is as follows.

```
TRAPZ  Trapezoidal numerical integration. Z = TRAPZ(X,Y) computes
The integral of Y with respect to X using the trapezoidal method.
X and Y must be vectors of the same length.
```

```
X =
```

```
theta = [0.4   1.2   1.7   1.1   3.1   1.28];
```

```
I =
```

========== End of the question paper ==========