University of Kelaniya - Sri Lanka
Center for Distance & Continuing Education
Bachelor of Science(General) External
Second Year second semester Examination - 2019 (2024 February)
(New Syllabus)
Faculty of Science

Applied Mathematics
AMAT 27572 - Numerical Methods II

No.of Questions: Five(05) No.of Pages: Three(03) Time: Two(2)hrs
Answer Four(04) Questions Only

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1. (a) (i) In the usual notations state L^1 norm and L^∞ norm of $\mathrm{x} = (x_1, x_2, ..., x_n)^T$ defined on $\mathbb{R}^n$.
   (ii) What are the properties should satisfied to be a norm?
   (iii) Verify that the function $||\,.\,||$ defined on $\mathbb{R}^n$ by
   $$||\mathrm{x}||_2 = \left(\sum_{k=1}^{n} |x_k|^2\right)^{\frac{1}{2}}$$
   is a norm on $\mathbb{R}^n$.

   (b) (i) In the usual notations state the Frobenius matrix norm $||A||_F$ and $||A||_2$ matrix norm.
   (ii) Find $||A||_F$ norm of the matrix $A = \begin{pmatrix} 2 & 1 \\ 4 & 5 \end{pmatrix}$.

   (c) (i) What is meant by ill conditioned system?
   (ii) Find the condition number of the following system of equations:
   $$x + 2y = 7$$
   $$x + 3y = 2\,.$$
   (iii) How do you determine the given system of equations is ill conditioned or well conditioned?

2. (a) (i) Starting with the Taylor series expansions of $f(t + \Delta t)$ and $f(t + 2\Delta t)$, derive the second order forward difference formula for $\dfrac{df}{dt}$ as
   $$\frac{-3f(t) + 4f(t + \Delta t) - f(t + 2\Delta t)}{2\Delta t}.$$
   (ii) Estimate the first derivative of $f(t) = 3t^2 - 4t + 7$ at $t = 1$ with a step size 0.5 by using the second order forward difference formula which is derived in part(i).

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(iii) Starting with the Taylor series expansions of $f(t+\Delta t)$ and $f(t-\Delta t)$, derive the second order centered difference formula for $\frac{d^2 f}{dt^2}$ as

$$\frac{f(t+\Delta t) - 2f(t) + f(t-\Delta t)}{(\Delta t)^2}.$$

(iv) Estimate the second derivative of $f(t) = 3t^2 - 4t + 7$ at $t = 1$ with a step size 0.2 by using the second order centered difference formula.

(b) Approximate $\int_0^1 e^{-x}dx$ to the third decimal place using the Simpson's $\frac{1}{3}$ Rule.

3. (a) Consider the following system of equations :

$$\begin{aligned} 8x + y + z &= 8 \\ 2x - 4y &= 4 \\ x + 3y + 5z &= 5 \end{aligned}$$

with the initial guess $(0, 0, 0)$.

(i) Determine whether the Jaccobi method is convergent for the above system of equations.

(ii) Find the solution to the system of equations using the Jaccobi method. Calculate the solution until the second iteration.

(b) (i) In the usual notations write the general formula for the Successive Over Relaxation (SOR) method.

(ii) Explain the Successive Over Relaxation method and the Successive Under Relaxation method using weighted value.

(iii) Solve the following linear system using the Successive Over Relaxation (SOR) method with the initial guess as $(1, 1, 1)$. Take the weighted value appropriately and calculate the solution until the second iteration.

$$\begin{aligned} 2x + z &= 2 \\ x + 3y &= 5 \\ x + 2y + 5z &= 3 \end{aligned}$$

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4. (a) Use the Heun's Method to solve $\frac{dy}{dx} = 1 - x + 2y$ with the initial condition $y(0) = 1$ for value of y at $x = 0.4$ by taking the step size $h = 0.2$.

   (b) Find the approximate solution for the differential equation $\frac{dy}{dx} = 2y - 3x$ with initial condition $y(0) = 1$ by the Picard's method to the 3rd approximation. Hence find the numerical solution at $x = 1$.

   (c) Consider the ordinary differential equation $\frac{dy}{dx} = y - 2x$ with the initial condition $y(0) = 1$. Find $y(0.6)$ using the Runge-Kutta method with step size $h = 0.6$.

5. (a) In the usual notations,

   (i) define Well posdness of the Initial Valued Problem.

   (ii) define the Lipschitz condition of a function.

   (iii) state the theorem for finding error bounds for the Euler's Method.

   (b) Consider the following Initial Valued Problem with the initial condition,

$$\begin{cases} f(t,y) = y - t^2 + 1\,; \quad 0 \leq t \leq 2 \\ y(0) = 0.5. \end{cases}$$

   (i) Show that the function f satisfies the Lipschitz condition.

   (ii) Find the error bound for the Euler's method to approximate the above IVP with $h = 0.2$. Consider the exact solution as $y(t) = (t+1)^2 - 0.5e^t$.

— End of Examination Paper —