University of Kelaniya – Sri Lanka
Centre for Distance & Continuing Education
Bachelor of Science (General) External
Second year second semester examination - 2023 (2026 July)
(New Syllabus)
Faculty of Science
Pure Mathematics
PMAT 27573- Ordinary Differential Equations

No. of Questions: Six **(06)** No. of Pages: Three **(03)** Time: **Two & half (2 ½) Hours.**

Answer **Five (05)** questions only.

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1. **(i)** Using existence and uniqueness theorem, determine whether the initial value problem $\frac{dy}{dx} = \frac{1}{x+y}$, $y(0) = 1$ has a unique solution. **[20 marks]**

(ii) Solve the initial value problem:

$\frac{dy}{dx} + 2xy = 2x,\ y(0) = 3.$ **[25 marks]**

(iii) Solve the following homogeneous differential equation:
$\frac{dy}{dx} = \frac{y}{x} + \tan\left(\frac{y}{x}\right)$ **[25 marks]**

(iv) Using the suitable substitution, transform the below differential equation into homogenous form and hence find the solution.
$\frac{y}{x}\frac{dy}{dx} + \frac{2(x^2+y^2)-1}{x^2+y^2+1} = 0$ **[30 marks]**

[Total 100 marks]

2. **(i)** **(a)** Determine whether the below differential equation is exact or not.

$(4xy + 3x^2)\, dx + (2y + 2x^2)\, dy = 0.$ **[20 marks]**

(b) Reduce the differential equation $y\sin 2x\, dx = (1 + y^2 + \cos 2x)\, dy$ into exact form and hence obtain the solution. **[30 marks]**

Continued...

(ii) **(a)** Using an appropriate substitution, transform the Bernoulli differential equation, in usual notation, given by

$$\frac{dy}{dx} + P(x)y = Q(x)y^n, \qquad n \neq 0,1.$$

into the form of linear differential equation

$$\frac{du}{dx} + (1-n)P(x)u = (1-n)Q(x).$$

[25 marks]

(b) Using part (a), or otherwise, solve the differential equation $\frac{dy}{dx} + \frac{2}{x}y = x^3y^2, x > 0.$ **[25 marks]**

[Total 100 marks]

3. **(i)** Consider the family of curves given by $(x-\lambda)^2 + y^2 = \lambda^2$, where λ is a parameter.

(a) Find the differential equation of the family of curves. **[10 marks]**

(b) Determine the differential equation of its orthogonal trajectories. **[10 marks]**

(c) Hence obtain the orthogonal trajectories. **[10 marks]**

(ii) Solve the differential equation $(x+y)^2\left(\frac{dy}{dx}\right) = a^2$, where a is a constant. **[20 marks]**

(iii) Suppose that a fish population $P(t)$ satisfies by the differential equation is

$$\frac{dP}{dt} = 0.4P\left(1 - \frac{P}{5000}\right).$$

If initially $P(t) = 500$, then determine the population

(a) after t years. **[30 marks]**

(b) when $t \to \infty$. **[20 marks]**

[Total 100 marks]

4. Solve the following Linear differential equations with constant coefficients:

(i) $(D^3 - 3D - 2)y = 540\, x^3e^{-x}$ **[25 marks]**

(ii) $(D^2 + 1)y = \sec x$ **[25 marks]**

(iii) $(D-1)^2y = xe^x \cos x$ **[20 marks]**

(iv) $(D^2 + 4)y = x \sin^2 x$ **[30 marks]**

[Total 100 marks]

Continued...

5. **(i)** Let $y = C_1u(x) + C_2v(x)$ be the general solution of $y'' + P(x)y' + Q(x)y = 0$.

Show that $y = A(x)u(x) + B(x)v(x)$ is a solution of the second order differential equation $y'' + P(x)y' + Q(x)y = R(x)$, where $A(x) = -\int \frac{vR}{W}dx, B(x) = \int \frac{uR}{W}dx$ and W being the Wronskian of u and v. **[30 marks]**

Hence, solve the differential equation $x^2\, y'' + xy' - y = x^2e^x$. **[25 marks]**

(ii) Find the general solution of the differential equation $y'' - y' - 6y = t^2e^{-3t} + cost$ using the method of undetermined coefficients. **[45 marks]**

[Total 100 marks]

6. **(i)** Use the substitution $u = \ln x$ to transform the ordinary differential equation with initial conditions

$$x^2\frac{d^2y}{dx^2} - 6x\frac{dy}{dx} + 12y = 0,\ \ y(1) = 0, y'(1) = 2$$

to

$$\frac{d^2y}{du^2} - 7\frac{dy}{du} + 12y = 0,\ \ y(0) = 0,\ \frac{dy}{du}(0) = 2.$$ **[35 marks]**

(ii) Solve the following differential equation by reducing into homogeneous form
$(1+x)^2\frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 4\,cos\,log(1+x)$.

[35 marks]

(iii) Reduce the equation $xyp^2 - (x^2 + y^2 - 1)p + xy = 0$ to Clairaut's form by using the substitution $x^2 = u$ and $y^2 = v$, and hence find the solution.
Here $p = \frac{dy}{dx}$.

[30 marks]

[Total 100 marks]

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