University of Kelaniya – Sri Lanka
Centre for Distance & Continuing Education
Bachelor of Science (General) External
Second year First semester examination - 2024 (New Syllabus)
2026, January
Faculty of Science
Pure Mathematics
PMAT 26562-Infinite Series

No. of Questions: **Five (05)** No. of Pages: **Two (02)** Time: **Two (02) hrs.**

Answer **Four (04)** Questions Only.

1) (i) What does it mean by saying that the sequence $\{a_n\}_{n=1}^{\infty}$ converges to a limit L, as $n \to \infty$? **(10 marks)**

(ii) Show that $\left\{\frac{n!}{n^n}\right\}_{n=1}^{\infty}$ converges. **(15 marks)**

(iii) Determine whether the sequence $\left\{\frac{2n^2-1}{n}\right\}_{n=2}^{\infty}$ is increasing, decreasing, not monotonic, bounded below, bounded above and/or bounded. **(30 marks)**

(iv) Let the sequence $\{a_n\}_{n=1}^{\infty}$ be defined recursively by $a_0 = 5,\ a_{n+1} = \frac{4}{7}a_n + 4$.

(a) Show that $\{a_n\}_{n=1}^{\infty}$ is increasing and bounded.

(b) Deduce that $\{a_n\}_{n=1}^{\infty}$ is convergent and find its limit.

(45 marks)

Total – [100 marks]

2) (i) Test the convergence of the following series by clearly mentioning the test you are using.

(a) $\sum_{n=0}^{\infty} 3\left(\frac{2}{5}\right)^n$ (b) $\sum_{n=5}^{\infty} \frac{6+8n+9n^2}{4+5n+n^2}$ **(20 marks)**

(ii) (a) State the Integral Test for a series using a function $f(x)$ defined on $[1, \infty)$. **(20 marks)**

(b) (I) By using the Integral Test, show that the series $\sum_{n=2}^{\infty} \frac{1}{(2n+7)^2}$ is convergent. **(20 marks)**

(II) Using $n = 5$, estimate the remainder R_5. **(10 marks)**

(III) Hence, obtained the improved estimate for the sum of the series. **(10 marks)**

(iii) Using the Ratio Test, determine whether or not the series $\sum_{n=1}^{\infty} \frac{9^n}{n\,(-2)^{n+1}}$ converges.

(20 marks)

Total – [100 marks]

Continued…

3) (i) Using the alternating series test, determine whether the series $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{\sqrt{n+3}}$ is convergent or divergent. **(25 marks)**

(ii) Using the Alternating Series estimation theorem, determine the minimum number of terms that must be summed, so that the approximation to the convergent series $\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}$ is accurate to within 0.0001. **(15 marks)**

(iii) Determine the positive integers k for which the series $\sum_{n=1}^{\infty}\frac{(n!)^2}{(kn)!}$ is absolutely converge. **(25 marks)**

(iv) Determine whether the following series are absolutely convergent, conditionally convergent or divergent.

(a) $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{\sqrt{n+1}+\sqrt{n}}$ (b) $\sum_{n=0}^{\infty}\frac{1+\sin(n)}{10^n}$ (c) $\sum_{n=1}^{\infty}\frac{(-1)^{n+2}}{4n-1}$ **(35 marks)**

Total – [100 marks]

4) (i) Test whether the following series are convergent or divergent by using the direct comparison or limit comparison Test.

(a) $\sum_{n=1}^{\infty}\frac{5}{(1+n^2)^2}$ (b) $\sum_{n=2}^{\infty}\frac{2}{\sqrt[3]{n}-1}$ (c) $\sum_{n=1}^{\infty}\frac{1}{\sqrt{4n^1+1}}$ **(45 marks)**

(ii) (a) Using the comparison test, estimate the error when approximate the sum of the series $\sum_{n=1}^{\infty}\frac{3}{n^3+2}$ by its first N terms. **(25 marks)**

(b) When error is 0.01, find the number of terms N. **(10 marks)**

(iii) Using the root test, determine the convergence of the series $\sum_{n=2}^{\infty}\frac{n^n}{(\ln(n))^n}$.

(20 marks)

Total – [100 marks]

5) (i) Determine the radius of convergence and the interval of convergence of the power series $f(x) = \sum_{n=1}^{\infty}\frac{(-1)^n x^n}{n^2 3^n}$. **(40 marks)**

(ii) Using the first three nonzero terms of the power series $f(x) = \sum_{n=0}^{\infty}\frac{2^n(n!)^2}{(2n)!}x^n$, estimate $\int_0^1\frac{f(x)-1}{x}dx$. **(20 marks)**

(iii) (a) In the usual notations, write down Taylor's inequality for a function $f(x)$.

(10 marks)

(b) Approximate the function $f(x) = \sqrt{x}$ by the Taylor polynomial of degree 3 near $x = 4$ .Then find the bound on the maximum error of the approximation on the interval [3,5] using the Taylor's inequality.

(30 marks)

Total – [100 marks]