



University of Kelaniya – Sri Lanka
Centre for Distance & Continuing Education
Bachelor of Science (General) External
Second year second semester examination - 2019 (2025 June)
(New Syllabus)
Faculty of Science
Pure Mathematics
PMAT 27572- Ordinary Differential Equations

No. of Questions: Five (05)

No. of Pages: Two (02)

Time: Two (02) Hours.

Answer **Four (04)** questions only.

1. (i) Determine the order, degree, and linearity of the following differential equations:

(a) $5x \frac{d^2y}{dx^2} + 3x^2 \frac{dy}{dx} - (\sin x) y = 0$

(b) $5t \frac{d^4y}{dt^4} + 3t^2 \left(\frac{dy}{dt}\right)^3 - (\sin t) y^6 = 0$ (20 marks)

(ii) Solve the linear differential equation $(x + 1) \frac{dy}{dx} + 2y = 1$. (25 marks)

- (iii) Consider the following initial value problem:

$$(2xy + y^2)dx + (x^2 + 2xy)dy = 0, y(1) = 2.$$

Show that the above equation is exact and hence, find its solution. (30 marks)

- (iv) Find the differential equation of the orthogonal trajectory for the family of curves given by the equation, $y^2 = 4\lambda(x + \lambda)$, where λ is a parameter. (25 marks)

[Total 100]

2. (i) Find the solution to the Bernoulli differential equation $\frac{dy}{dx} + 2y = xy^{-2}$.

(30 marks)

(ii) Solve the homogeneous differential equation, $\frac{dy}{dx} = \frac{x^2 + xy + y^2}{x^2}$. (30 marks)

- (iii) Solve the following differential equation by reducing to homogeneous form:

$$(2x + y + 1)dx + (4x + 2y - 1)dy.$$
 (40 marks)

[Total 100]

continued...

3. Solve the following differential equations:

- (i) $(D^3 - D^2 - 6D)y = x^3 + 1$
- (ii) $(D^2 + 9)y = \sin 2x$
- (iii) $(D^2 - 4D + 1)y = e^{2x} \sin x$
- (iv) $(D^2 - 4D + 4)y = e^x + x^2 + \cos 2x$

(25 marks each)
[Total 100]

4. (i) Let $y = C_1 u(x) + C_2 v(x)$ be the general solution of $y'' + P(x)y' + Q(x)y = 0$.

(a) Show that $y = A(x)u(x) + B(x)v(x)$ is a solution of

$$y'' + P(x)y' + Q(x)y = R(x),$$

where $A(x) = -\int \frac{vR}{W} dx$, $B(x) = \int \frac{uR}{W} dx$ and W being the Wronskian of u

and v .

(25 marks)

(b) Using part (a), solve the following differential equation:

$$y'' - 2y' + y = e^{2x}.$$

(35 marks)

(ii) Find the general solution of the following differential equation using the method of undetermined coefficients:

$$(D^2 - 10D + 29)y = -8e^{5x} \sin 2x.$$

(40 marks)

[Total 100]

5. (i) Using a substitution $z = \log x$, show that $\frac{dy}{dx} = \frac{1}{x} \frac{dy}{dz}$ and $\frac{d^2y}{dx^2} = \frac{1}{x^2} \left(\frac{d^2y}{dz^2} - \frac{dy}{dz} \right)$.

Hence, solve the equation $x^2 D^2 y - 3x Dy + 5y = x^2 \sin \log x$. (60 marks)

(ii) By using suitable substitution reduce the ordinary differential equation

$$(1+x)^2 \frac{d^2y}{dx^2} + (1+x) \frac{dy}{dx} + y = 4 \cos \log(1+x)$$

to homogeneous form and hence find the solution.

(40 marks)

[Total 100]

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