University of Kelaniya – Sri Lanka
Centre for Distance & Continuing Education
Bachelor of Science (General) External
Second year first semester examination (Repeat) - 2021
(New Syllabus)
2025 June
Faculty of Science
Pure Mathematics
PMAT 26562(R)-Infinite Series

No. of Questions: **Five (05)** No. of Pages: **Three (03)** Time: **Two (02) hrs.**

Answer **Four (04)** Questions Only.

1) (i) Assume that in the usual notations, $\lim_{n\to\infty} s_n = a$ and $\lim_{n\to\infty} t_n = b$. Using the $\varepsilon - N$ definition of the limit of a convergent sequence, prove that $\lim_{n\to\infty}(s_n + t_n) = a + b$.

(20 marks)

(ii) Determine wheather the sequence $\left\{\frac{\ln(n+2)}{\ln(1+4n)}\right\}_{n=1}^{\infty}$ converges or diverges. If it converges, find its limit. **(15 marks)**

(iii) Let the sequence $\{a_n\}_{n=1}^{\infty}$ be defined recursively by $a_1 = 0,\ a_{n+1} = \frac{1}{3}(a_n + 3)$ for all $n \in \mathbb{N}$.

(a) Show that $a_n < 3$ for all $n \in \mathbb{N}$.
(b) Show that $\{a_n\}_{n=1}^{\infty}$ is increasing.
(c) Deduce that $\{a_n\}_{n=1}^{\infty}$ is convergent and find its limit.

(35 marks)

(iv) Determine whether the sequence $\left\{\frac{4-n}{2n+3}\right\}_{n=1}^{\infty}$ is increasing, decreasing, monotonic, bounded below, bounded above and/or bounded.

(30 marks)

Total – [100 marks]

Continued...

2) (i) (a) State the p-series test. **(10 marks)**

(b) Determine whether the series $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right) + \frac{1}{n^2}$ is convergent? **(20 marks)**

(ii) (a) State the Integral Test for a series using a function defined on $[1, \infty)$.

(15 marks)

(b) Consider the series $\sum_{n=1}^{\infty} \frac{1}{(n+1)^2}$;

(α) Using the Integral Test, show that the above series is convergent.

(β) Estimate the bounds of the remainder R_5 using the partial sum of the first five terms S_5.

(γ) Using part (β), find an improved estimate for the sum of the series.

(δ) Find the smallest value of n such that the approximation error is within 0.0001? **(55 marks)**

Total – [100 marks]

3) (i) State the Alternating Series Test for the convergence of the series $\sum(-1)^n b_n$. **(15 marks)**

(ii) (a) Using the alternating series test, show that the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{10n^2}$ is convergent. **(25 marks)**

(b) Using the Alternating Series Estimation Theorem, estimate the minimum number of terms needed to achieve an approximation error less than 0.0005. **(20 marks)**

(iii) Determine by stating any test you use whether the following series are absolutely convergent, conditionally convergent or divergent.

(a) $\sum_{n=1}^{\infty} \frac{(-1)^{n-3}}{\sqrt{n}}$ (b) $\sum_{n=0}^{\infty} \frac{(-2)^{3n}}{5^n}$ (c) $\sum_{n=1}^{\infty} \frac{\sin(3n)}{n^6+1}$ **(40 marks)**

Total – [100 marks]

4) (i) (a) State the comparison test using two positive series represented by $\sum a_n$ and $\sum b_n$. **(20 marks)**

(b) Using 50 terms, and by using the comparison test, estimate the error involved when estimating the sum of the series $\sum_{n=1}^{\infty} \frac{1}{n^3+1}$. **(20 marks)**

(ii) Determine whether the following series are convergent or divergent by using the direct comparison or limit comparison Test.

(a) $\sum_{n=0}^{\infty} \frac{1}{3^n - n}$ (b) $\sum_{n=1}^{\infty} \frac{n}{n^2 - \cos^2(n)}$ (c) $\sum_{n=1}^{\infty} \frac{4n^2 - n}{n^3+9}$ **(40 marks)**

Continued...

(iii) Using the root test, determine whether the series $\sum_{n=4}^{\infty} \frac{(-5)^{1+2n}}{2^{5n-3}}$ is convergent or divergent. **(20 marks)**

Total – [100 marks]

5) (i) Determine the radius of convergence and the interval of convergence of the power series $\sum_{n=1}^{\infty}(-1)^n \frac{1}{n10^n}(x-2)^n$ **(40 marks)**

(ii) Starting from a known geometric series, find a Maclaurin series representation for $\ln(1+x)$.
Hence, evaluate
(a) $\ln\left(\frac{1+x}{1-x}\right)$ (b) $\int_0^1 \frac{\ln(1+x)}{x}\,dx$

(30 marks)

(iii) (a) In the usual notation, write down Taylor's inequality for a function $f(x)$. **(10 marks)**

(b) Find the degree three Taylor Polynomial for the function $f(x) = \sqrt{x}$ about the center 1.
Also, find the remainder term using the Taylor's inequality.

(20 marks)

Total – [100 marks]

.............................**END OF PAPER**.............................