UNIVERSITY OF KELANIYA - SRI LANKA

Center for Distance & Continuing Education
Bachelor of Science (General) External
Second year Second semester examination - 2019
(New Syllabus)
2024 February
Faculty of Science

Pure Mathematics
PMAT 27583 - Functionsof Several Variables

No.of Pages: Nine(09) Time: $2\frac{1}{2}$hrs

Instruction for candidates: This paper consists two parts.
Part I: This part is compulsory. Answer ALL the multiple-choice questions. Show all your work in the provided space to get full marks. *Circle your answer.*
Part II: Three (03) questions should be attempted from the Part II. Use the given booklet to answer the questions.

The use of calculator, formula sheet and/or any other electronic device is not allowed.

Student number: ....

Part I

[20 marks for each correct answer – Total for Part I is 200 marks]

1. The domain of the function $f(x,y) = \dfrac{1}{x} + \sqrt{y+4} - \sqrt{x+1}$ is

(a) $x \neq 0, y \geq -4, x \geq -1$
(b) $x \neq 0, y \neq -4, x \neq -1$
(c) $y \geq -4, x \geq -1$
(d) $y+4=0, x \neq 4, x \neq 0$
(e) $x \neq 0, y \neq 4$

Continued...

2. Let $\theta = \arccos(3/5)$. Find the directional derivative $D_{\vec{u}}f(x,y)$ of $f(x,y) = x^2 - xy + 3y^2$ in the direction of $\vec{u} = (\cos\theta)\hat{\imath} + (\sin\theta)\hat{\jmath}$.

(a) 0 (b) $\dfrac{2x+21y}{5}$ (c) ∞ (d) $\dfrac{2x+5y}{3}$ (e) $\dfrac{2y}{5}$

3. Convert the point $(8, \pi/3, \pi/6)$ from spherical coordinates to cylindrical coordinates.

(a) $\left(2, \dfrac{\pi}{3}, \sqrt{3}\right)$ (b) $\left(\dfrac{1}{3}, -4, \dfrac{1}{2}\right)$ (c) $\left(4, \dfrac{\pi}{3}, 4\sqrt{3}\right)$

(d) $\left(2, \dfrac{\pi}{3}, \sqrt{3}\right)$ (e) $\left(2, \dfrac{\pi}{3}, 2\sqrt{3}\right)$

Continued ...

4. Find the equation of the plane tangent to the surface $x^2 + y^2 + z^2 = 14$ at the point $(2, 1, 3)$.

(a) $2x + y + 3z - 14 = 0$
(b) $x^2 + y + 3z^2 - 14 = 0$
(c) $2(x-2) + (y-1) + 3(z-3) - 14 = 0$
(d) $2x(x-1) + 2y(y-1) + 2z(z-3) = 0$
(e) $2(x-2) + (y-1) + 3(z-3) + 14 = 0$

5. A critical point of f is a point (a, b) in the domain of f if

(a) both $\frac{\partial f}{\partial x}(a,b) = 0$ and $\frac{\partial f}{\partial y}(a,b) = 0$ or at least one of the partial derivatives does not exit.
(b) both $\frac{\partial f}{\partial x}(a,b) = 0$ and $\frac{\partial f}{\partial y}(a,b) = 0$.
(c) one of the partial derivatives does not exit.
(d) both $\frac{\partial f}{\partial x}(a,b) \neq 0$ and $\frac{\partial f}{\partial y}(a,b) \neq 0$ or at least one of the partial derivatives does not exit.
(e) None of the above.

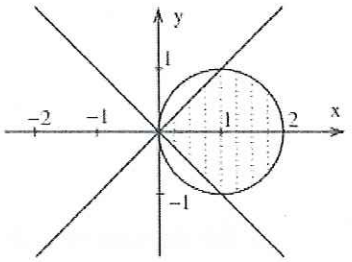
Continued...

6. Find the relative extrema of $f(x, y) = x^2 + y^2 - xy - x - 2$.

(a) $\left(\frac{1}{2}, 2\right)$ (b) $(0, 0)$ (c) $\left(\frac{1}{2}, 4\right)$ (d) $(1, 0)$ (e) $\left(\frac{2}{3}, \frac{1}{3}\right)$

7. Set up the double integral in polar coordinates (**Do not compute it!**) for finding the area of the region defined by

$$D = \begin{cases} -x \leq y \leq x \\ (x-1)^2 + y^2 \leq 1 \end{cases}$$



(a) $\displaystyle\int_{\theta=-\pi/4}^{\theta=\pi/4} \int_{r=0}^{r=1} drd\theta$

(b) $\displaystyle\int_{\theta=-\pi/2}^{\theta=\pi/2} \int_{r=0}^{r=\cos\theta} rdrd\theta$

(c) $\displaystyle\int_{\theta=-\pi/4}^{\theta=\pi/4} \int_{r=0}^{r=2\cos\theta} rdrd\theta$

(d) $\displaystyle\int_{\theta=-\pi/4}^{\theta=\pi/4} \int_{r=0}^{r=2\cos\theta} drd\theta$

(e) $\displaystyle\int_{\theta=-\pi/2}^{\theta=\pi/2} \int_{r=0}^{r=2\cos\theta} rdrd\theta$

Continued...

8. Reverse the order of the following integral

$$\int_0^3 \int_{y^2}^9 f(x,y)dxdy.$$

(a) $\int_0^3 \int_0^{\sqrt{x}} f(x,y)dxdy$ (b) $\int_0^9 \int_0^{\sqrt{x}} f(x,y)dydx$ (c) $\int_0^{\sqrt{x}} \int_0^9 f(x,y)dydx$

(d) $\int_0^9 \int_{-\sqrt{x}}^0 f(x,y)dydx$ (e) $\int_{-\sqrt{x}}^0 \int_0^9 f(x,y)dydx$

9. Find the volume V under the plane $z = 8x + 6y$ over the region $R = [0,1] \times [0,2]$.

(a) $1 - \frac{\sqrt{2}}{2}$ (b) 0 (c) 20 (d) 1 (e) $\frac{16\pi}{3}$

Continued...

10. Evaluate $\int_0^{2\pi} \int_0^{\pi} \sin(x + y)dxdy$

(*a*) $\sqrt{2}$ (*b*) 100 (*c*) 0 (*d*) π (*e*) 2π

... End of Part I ...

Final Exam - PMAT 27583 - Part II

Answer only 3 questions.

1. (a) Identify and sketch the level curves (contours) for the function

$$-x + 2y^2 + 4z = 0.$$

[10 Marks]

(b) Find the following limits, if they exist.

i. $\lim_{(x,y)\to(0,0)} \dfrac{xy}{x^2+y^2}$ ii. $\lim_{(x,y)\to(0,0)} \dfrac{x^4y^4}{x^2+y^2}$

[40 Marks]

(c)

i. Define the continuity of the two variable function at the point (a, b).

[10 Marks]

ii. A function $f(x, y)$ is defined by

$$f(x,y) = \begin{cases} \dfrac{x^2y^2}{x^4+y^4-x^2y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0). \end{cases}$$

Discuss the continuity of $f(x, y)$ at the point $(0, 0)$.

[40 Marks]

2. (a) Find y' of the given implicit defined function

$$\sin(x^2y^2) + y^3 = x + y.$$

[20 Marks]

(b) Determine whether the function

$$z = \frac{1}{2}e^{x+y}$$

is solution of the Laplace's equation $z_{xx} + z_{yy} = 0$.

[20 Marks]

(c) Compute the slope of the tangent plane to the graph of $f(x, y) = \tan^{-1}(\frac{x}{y})$ at the given point $P_0(1, \sqrt{3})$ in the direction parallel to

i. the $xz-$plane ii. the $yz-$plane.

[20 Marks]

(d) A certain country's production in the early years following World War II is described by the function

$$f(x, y) = 30x^{2/3}y^{1/3}$$

when x units of labor and y units of capital were used.

i. The first partial derivative f_x , f_y is called the marginal productivity of labor and marginal productivity of capital respectively. Compute f_x and f_y.

[20 Marks]

ii. Find the marginal productivity of labor and the marginal productivity of capital when the amount expended on labor and capital was 125 units and the 27 units, respectively.

[20 Marks]

3. (a) Consider the function

$$f(x, y) = x^3 - 3x - y^2 + 4y.$$

i. Find the relative extrema of f.

[30 Marks]

ii. Determine whether the function has a relative minimum, maximum or saddle point at each critical point.

[40 Marks]

(b) By using method of Lagrange multipliers, find the relative minimum of the function

$$f(x, y) = 2x^2 + y^2$$

subject to the constraint $x + y = 1$. [30 Marks]

4. (a) .

i. Sketch the polar rectangular region

$$R = \{(r, \theta)|1 \leq r \leq 3, 0 \leq \theta \leq \pi\}.$$

[10 Marks]

ii. Evaluate the integral $\iint\limits_R 3x dA$ over the region R defined in part i.

[20 Marks]

(b) Consider ,

$$\int_0^1 \int_{-x^2}^{x^2} dy\, dx$$

i. evaluate the integral. [10 Marks]

ii. Re do part i by changing the order of the integral. [20 Marks]

(c) Evaluate the following integral

$$I = \int_0^3 \int_0^{\sqrt{9-x^2}} x \, dy \, dx$$

[40 Marks]

.... End of question paper ....