University of Kelaniya – Sri Lanka
Centre for Distance & Continuing Education
Bachelor of Science (General) External
Second year second semester examination - 2019 (2024 February)
(New Syllabus)
Faculty of Science
Pure Mathematics
PMAT 27572- Ordinary Differential Equations

No. of Questions: Five **(05)** No. of Pages: Two **(02)** Time: **Two (02) Hours.**

Answer **Four (04)** questions only.

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1. (i) Using existence and uniqueness theorem, determine whether the initial value problem $y' = y^2,\ y(0) = 1$ has a unique solution.

(ii) Find the solution of the initial value problem, $\frac{dy}{dx} + y\,tanx = e^{2x}cosx,\ y(0) = 2.$

(iii) Consider the following initial value problem:

$$(x^2 + 3y^2)\frac{dy}{dx} + 2xy = 0, \quad y(1) = 1.$$

Show that the above equation is exact and hence, find the solution.

(iv) Find the differential equation of the family of curves given by the equation, $x^2 - y^2 + 2\lambda xy = 1$, where λ is a parameter. Hence, obtain the differential equation of its orthogonal trajectories.

2. (i) Show that the differential equation $xy' + y = -3xy^2$ is in the form of Bernoulli equation and hence, find the solution.

(ii) Reduce the following differential equation into homogeneous form and hence find the solution: $\frac{dy}{dx} = \frac{x+2y-3}{2x+y-3}$

continued...

3. Solve the following differential equations:

(i) $(D^3 + 8)y = x^4 + 2x + 1$

(ii) $(D^2 - 5D + 6)y = e^{4x}\,(x^2 + 9)$

(iii) $(D^2 - 1)y = x^2 \cos x$

(iv) $(D^2 - 1)y = xe^x + \frac{1}{2} + \frac{\cos 2x}{2}$

4. (i) Let $y = C_1 u(x) + C_2 v(x)$ be the general solution of $y'' + P(x)y' + Q(x)y = 0$.

Show that $y = A(x)u(x) + B(x)v(x)$ is a solution of $y'' + P(x)y' + Q(x)y = R(x)$,

where $A(x) = -\int \frac{vR}{W}dx,\ B(x) = \int \frac{uR}{W}dx$ and W being the Wronskian of u and v.

Hence, solve the following differential equation:

$$y'' - 3y' + 2y = e^{-x}.$$

(ii) Find the general solution of the differential equation $(D^2 + 2D + 2)y = x^2 + \sin x$ using the method of undetermined coefficients.

5. (i) Consider the second order liner differential equation with variable coefficients

$a_2 x^2 \frac{d^2y}{dx^2} + a_1 x \frac{dy}{dx} + a_0 y = F(x)$, where a_0, a_1, a_2 are constants.

Using a substitution $z = \log x$, show that $\frac{dy}{dx} = \frac{1}{x}\frac{dy}{dz}$ and $\frac{d^2y}{dx^2} = \frac{1}{x^2}\left(\frac{d^2y}{dz^2} - \frac{dy}{dz}\right)$.

Hence, convert the differential equation into the form,

$$a_2 \frac{d^2y}{dx^2} + (a_1 - a_2)\frac{dy}{dx} + a_0 y = Q(e^z).$$

(ii) Using part(i), solve the following differential equation:

$$x^2 \frac{d^2y}{dx^2} + 7x\frac{dy}{dx} + 13y = \log x.$$

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