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University of Kelaniya-Sri Lanka

බාහිරවිභාග අංශය

External Examinations Branch

විද්‍යා පීඨය - Faculty of Science

විද්‍යාවේදී (සාමාන්‍ය) උපාධි ප්‍රථම පරීක්ෂණය (බාහිර) - 2008

2010 ඔක්තෝබර්

Bachelor of Science (General) Degree First Examination (External) 2008

October -2010

Statistics & Computer Science

STCS E 1015 – Probability and Statistics

No. of Questions: **Eight (08)** No. of Pages: **Four (04)** Time: **Three (3) hours**

Answer Six (06) Questions Only

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1. (a) (i) Prove that for any two events A and B

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

Extend this result for any three events A, B and C.

(ii) When a person visits a dentist, the probabilities that he will have his teeth cleaned, a cavity filled and removed the enamel of a tooth are 0.44, 0.24 and 0.21 respectively. The probabilities that he will have his teeth cleaned and cavity filled, his teeth cleaned and removed the enamel of a tooth, his cavity filled and removed the enamel of a tooth are 0.08, 0.11 and 0.07 respectively. The probability that he will have all three, that is, his teeth cleaned, cavity filled and removed the enamel of a tooth is 0.03.

Calculate the probability that a person visiting this dentist will have at least one of these activities.

(b) A consulting firm rent 60% of cars from company A_1, 30% from company A_2 and 10% from company A_3. Probability of a car requiring a tune-up given that it is rented from A_1 is 0.09. These probabilities for A_2 and A_3 are 0.2 and 0.06 respectively. Calculate the probability that the car was rented from company A_2 given that the car rented by the consulting firm requires a tune-up.

2. (a) (i) In the usual notation, write down the probability density function for a binomial distribution.

(ii) Determine the moment generating function of the above binomial distribution. Hence find the mean and the variance of the distribution.

(b) When a machine is functioning properly, 3% of the parts produced are defective. Suppose that a random sample of 12 parts has been observed for defects. Model this situation as a binomial distribution.

Hence compute the probability of having

(i) more than 1 defective part in the selected sample.

(ii) more than 1 defective part in each of two such samples.

Contd...

3. (a) (i) In the usual notation, write down the probability density function for a normal distribution with parameters μ and σ.

(ii) Determine the mean and the variance of the normal distribution given above.

(b) Finance division of a particular bank has complied data on the pay-back duration of credit card accounts and is assume to be normally distributed with mean 28 days and standard deviation 8 days.

(i) Calculate the percentage of accounts whose pay-back durations are in between 20 and 40 days.

(ii) Bank administration is interested in sending reminder letters to the accounts in the largest 15% pay-back duration. How many days an account should be without pay-back before a reminder letter is received.

(iii) The bank administration would like to give a discount to the accounts which pay their balance on or before 22 days. Compute the percentage of the accounts which receives this discount.

4. Bivariate random variable (X, Y) have the joint distribution defined by the following table of probabilities:

		X		
		1	2	3
Y	2	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{12}$
	3	$\frac{1}{6}$	0	$\frac{1}{6}$
	4	0	$\frac{1}{3}$	0

Find

(i) the marginal probability mass functions of X and Y,

(ii) $P(X = Y)$,

(iii) $P(X = 2 \text{ or } Y = 4)$,

(iv) $P(X + Y \leq 4)$,

(v) $E(Y \mid X = 3)$.

5. Random variables X and Y have joint probability density function $f_{X,Y}(x, y)$ given by

$$f_{X,Y}(x, y) = \begin{cases} 8xy & ; 0 < x < y < 1 \\ 0 & ; \text{Otherwise} \end{cases}$$

Find

(i) the marginal probability density function of X,

(ii) the conditional probability density function of X given $Y = y$,

(iii) $P(X \leq \frac{1}{4} \mid Y = \frac{2}{3})$,

(iv) $P(X \leq \frac{1}{2} \mid \frac{1}{2} < Y < 1)$.

6. (a) (i) Define what is meant by the correlation coefficient.

(ii) Let $U = kX$, $W = kY$; where k is a constant.
Show that the correlation coefficient of U and W is the same as the correlation coefficient of X and Y.

(b) (i) If $X+Y$ and $X+Y$ are uncorrelated, show that $Var(X) = Var(Y)$.

(ii) Suppose that the random variable X has the following discrete probability distribution:

x	-1	0	1
$P(X=x)$	$1/3$	$1/3$	$1/3$

Compute $Var(X)$, $Var(X^2)$ and the correlation coefficient of X and X^2

7. (a) Random variable X have $Beta(a,b)$ distribution with probability density function $f(x) = \frac{1}{B(a,b)} x^{a-1}(1-x)^{b-1}$, for $0 < x < 1$; where $B(a,b) = \frac{\Gamma(a).\Gamma(b)}{\Gamma(a+b)}$.

Show that $E(X) = \frac{a}{a+b}$ and $V(X) = \frac{ab}{(a+b+1)(a+b)^2}$

(b) X and Y be independent random variables, each having a Gamma distributions with parameters (a,α) and (b,α), respectively. Consider the transformations $U = \frac{X}{X+Y}$ and $V = X+Y$.

Find the joint density of U and V .

Hence

(i) Show that U and V are independent.

(ii) Obtain the marginal distributions of U and V . Identify these distributions.

[Hint: You may assume the distribution function of a gamma distribution with parameters (n,θ) as $f_X(x) = \frac{1}{\Gamma(n)} \theta^n x^{n-1} e^{-\theta x}$, for $x > 0$]

8. (a) (i) Let $\{X_1, X_2, ..., X_n\}$ be a random sample drawn from a continuous distribution with cumulative distribution function F_X and probability density function f_X. Let Y_n denote the n^{th} order statistic.

Show that the probability density function of Y_n is given by $g_{Y_n}(y) = n[F_X(y)]^{n-1} f_X(y)$

(iii) Let $\{X_1, X_2, X_3\}$ be a random sample drawn from the uniform distribution with probability density function given by $f_X(x) = \begin{cases} 1 & ; 0 < x < 1 \\ 0 & ; otherwise \end{cases}$

and $Y = X_{(3)}$; where $X_{(3)}$ is the 3rd order statistic.

Show that $P\left[Y > \frac{1}{4}\right] = \frac{63}{64}$.

(b) Joint density of the r^{th} and s^{th} ($r < s$) order statistics of a random sample $\{X_1, X_2, ...X_n\}$ from a population with cumulative distribution function F_X and probability density function f_X, is given by,

$$g_{X_{(r)},X_{(s)}}(x,y) = \frac{n!}{(r-1)!(s-r-1)!(n-s)!}\left[F_X(x)\right]^{r-1}\left[F_X(y)-F_X(x)\right]^{s-r-1}\left[1-F_X(y)\right]^{n-s} f_X(x).f_X(y)$$

Show that the joint density function of $X_{(1)}$ and $X_{(n)}$ for a uniform population in the interval (0,1) is,

$$g_{X_{(1)},X_{(n)}}(x,y) = n(n-1)(y-x)^{n-2}, \; 0 < x < y < 1.$$

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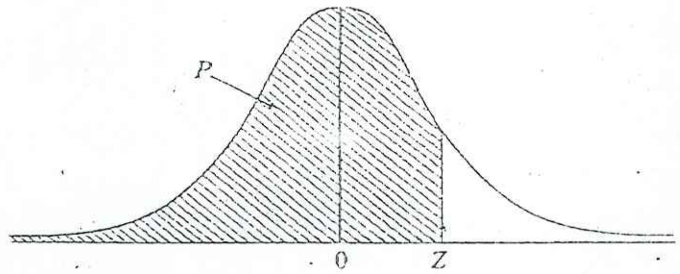
Z	P	Z	P	Z	P
-4.00	0.00003	-1.00	0.1587	1.05	0.8531
-3.50	0.00023	-0.95	0.1711	1.10	0.8643
-3.00	0.0014	-0.90	0.1841	1.15	0.8749
-2.95	0.0016	-0.85	0.1977	1.20	0.8849
-2.90	0.0019	-0.80	0.2119	1.25	0.8944
-2.85	0.0022	-0.75	0.2266	1.30	0.9032
-2.80	0.0026	-0.70	0.2420	1.35	0.9115
-2.75	0.0030	-0.65	0.2578	1.40	0.9192
-2.70	0.0035	-0.60	0.2743	1.45	0.9265
-2.65	0.0040	-0.55	0.2912	1.50	0.9332
-2.60	0.0047	-0.50	0.3085	1.55	0.9394
-2.55	0.0054	-0.45	0.3264	1.60	0.9452
-2.50	0.0062	-0.40	0.3446	1.65	0.9505
-2.45	0.0071	-0.35	0.3632	1.70	0.9554
-2.40	0.0082	-0.30	0.3821	1.75	0.9599
-2.35	0.0094	-0.25	0.4013	1.80	0.9641
-2.30	0.0107	-0.20	0.4207	1.85	0.9678
-2.25	0.0122	-0.15	0.4404	1.90	0.9713
-2.20	0.0139	-0.10	0.4602	1.95	0.9744
-2.15	0.0158	-0.05	0.4801	2.00	0.9772
-2.10	0.0179	0.00	0.5000	2.05	0.9798
-2.05	0.0202	0.05	0.5199	2.10	0.9821
-2.00	0.0228	0.10	0.5398	2.15	0.9842
-1.95	0.0256	0.15	0.5596	2.20	0.9861
-1.90	0.0287	0.20	0.5793	2.25	0.9878
-1.85	0.0322	0.25	0.5987	2.30	0.9893
-1.80	0.0359	0.30	0.6179	2.35	0.9906
-1.75	0.0401	0.35	0.6368	2.40	0.9918
-1.70	0.0446	0.40	0.6554	2.45	0.9929
-1.65	0.0495	0.45	0.6736	2.50	0.9938
-1.60	0.0548	0.50	0.6915	2.55	0.9946
-1.55	0.0606	0.55	0.7088	2.60	0.9953
-1.50	0.0668	0.60	0.7257	2.65	0.9960
-1.45	0.0735	0.65	0.7422	2.70	0.9965
-1.40	0.0808	0.70	0.7580	2.75	0.9970
-1.35	0.0885	0.75	0.7734	2.80	0.9974
-1.30	0.0968	0.80	0.7881	2.85	0.9978
-1.25	0.1056	0.85	0.8023	2.90	0.9981
-1.20	0.1151	0.90	0.8159	2.95	0.9984
-1.15	0.1251	0.95	0.8289	3.00	0.9986
-1.10	0.1357	1.00	0.8413	3.50	0.99977
-1.05	0.1469			4.00	0.99997

සම්මත ප්‍රමත ව්‍යාප්තිය
நியம செவ்வன் பரம்பல்
The Standardised Normal Distribution



මධ්‍යන්‍යය ශුන්‍යයද, සම්මත අපගමනය 1 ද වන සම්මත ප්‍රමත ව්‍යාප්තියක් පහත වගුකර ඇත. Z හි සෑම අගයකටම සම්මත ප්‍රමිති සමානුපාතය (ව්‍යාප්තියේ P නම් සමානුපාතය Z ට වඩා අඩුවීමට) මෙම වගුව භාවිතා කර සෙවිය හැක.

මධ්‍යන්‍යය μ ද සම්මත අපගමනය σ (විචලතාව σ^2 වේ) ද වන ප්‍රමත ව්‍යාප්තියක, යම්කිසි X අගයකට වඩා අඩු සමානුපාතය ලබාගැනීමට, $Z = (X - \mu)/\sigma$ සූත්‍රය ආධාරයෙන් Z ගණනය කර එම Z ට අනුරූප සමානුපාතය වගුවෙන් ලබාගත හැක.

பூச்சிய இடையையும், ஓரலகு நியம விலகலையும் கொண்ட செவ்வன் பரம்பலொன்று அட்டவணைப்படுத்தப்பட்டுள்ளது. நியம செவ்வன் விலகல் Z இன், ஒவ்வொரு பெறுமதிக்கும் Z இலும் குறைந்த, பரம்பலின் விகிதம் P தரப்பட்டுள்ளது. μ இனை இடையாகவும், σ^2 இனை மாறற்றிறனாகவும் கொண்டவொரு செவ்வன் பரம்பலுக்கு தரப்பட்ட பெறுமதி X இலும் குறைந்த, பரம்பலின் விகிதமானது $Z = (X - \mu)/\sigma$ இனைக் கணித்து இவ் Z பெறுமதிக்கு ஒத்த விகிதத்தை வாசித்துப் பெறப்பட்டது.

The distribution tabulated is that of the normal distribution with mean 0 and standard deviation 1. For each value of Z, the standardized normal deviate, (the proportion P, of the distribution less than Z) is given. For a normal distribution with mean μ and variance σ^2 the proportion of the distribution less than some particular value X is obtained by calculating $Z = (X - \mu)/\sigma$ and reading the proportion corresponding to this value of Z.