UNIVERSITY OF KELANIYA - SRI LANKA

Bachelor of Science (General) Degree First Examination (External) – February 2016

PMAT E 1025 - Discrete Mathematics I

No. of Questions : Eight (08) **No. of Pages : Four (04)** **Time : Three (03) hrs**

Answer <u>Six (06)</u> Questions only.

1.

Let A and B be any two subsets of a universal set E.

(a) Show by using the first principles that

(i) $(A \cap B)' = A' \cup B'$

(ii) $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$

(b) Show, by using the algebra of sets, that

$$(B - C) - (A - C) = A - (B \cup C).$$

(c) Let $\mathcal{P}(X)$ denote the power set of X. Prove that if $A \subseteq B$ then $\mathcal{P}(A) \subseteq \mathcal{P}(B)$.

2.

(a) Let R be a relation defined on a set A.

Define

(i) what is meant by saying that R is an equivalence relation on A and

(ii) the equivalence class $[x]$ of $x \in A$.

(b) Let R be a relation defined on $\mathbb{Z}^+ \times \mathbb{Z}^+$ as follows:

$(a, b)R(c, d)$ if and only if $a \cdot d = b \cdot c$

(i) Show that R is an equivalence relation.

(ii) Find the equivalence class of $(2,3)$ and represent graphically.

(c) Let $A = \{1,2,3\}$. Define a relation R on A such that R is reflexive and transitive but not symmetric.

Continued...,

3.

(a) Let $f: \mathbb{R} \longrightarrow \mathbb{R}$ be the function defined by $f(x) = \frac{x}{x^2+1}$. Determine whether f is

(i) one-to-one

(ii) onto

(b) Let $f: X \to Y$ and $g: Y \to Z$ be two one-to-one functions. Show that $g \circ f$ is also one-to-one.

(c) Suppose that the function $f: \mathbb{R} \longrightarrow \mathbb{R}$ is defined by $f(x) = 3x - 1$ and that the function $g: \mathbb{R} \longrightarrow \mathbb{R}$ is defined by $g(x) = ax + b$ where a and b are constants. Find constants a and b such that $(g \circ f)(x) = x$.

4. (a) If A and B are two invertible matrices of the same order n, then show that

$$adj(AB) = (adjB)(adjA).$$

(b) Suppose A and B are two non-singular symmetric matrices. If A and B are commutative then show that $A^{-1}B$ is a symmetric matrix.

(c) Using induction, prove that for all integers $n \geq 1$,

$$(A_1 + A_2 + \cdots + A_n)^T = A_1^T + A_2^T + \cdots + A_n^T.$$

(d) Let $A = \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix}$.

Show that $A^6 = I$. Hence find A^{2017}.

5. (a) Prove that the value of the real determinant $|A| = |a_{ij}|_{n \times n}$ in standard notation, is unchanged when the elements of any row are added k times the corresponding elements of another row, where k is a real constant.

(b) Solve the equation

$$\begin{vmatrix} a-x & b-x & c \\ a-x & c & b-x \\ a & b-x & c-x \end{vmatrix} = 0,$$

where a, b and c are real numbers.

(c) Using the properties of determinants simplify

$$\Delta = \begin{vmatrix} a & b & b & b \\ a & b & a & a \\ a & a & b & a \\ b & b & b & a \end{vmatrix}.$$

Continued...,

6. (a) Consider the system of linear equations

$$x + y = 2$$
$$y - z = 0$$
$$x + 2w = 1$$
$$x + y + z + \lambda w = \mu, \text{ where } \lambda \text{ and } \mu \text{ are parameters.}$$

Find the conditions on λ and μ such that the system

(i) has a unique solution,

(ii) has infinitely many solution, and

(iii) has no solution.

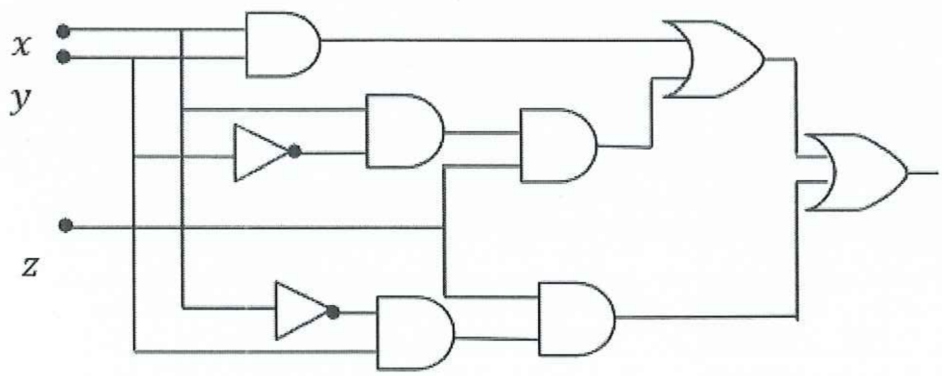
(b) Let $A\underline{x} = \underline{0}$ be the matrix representation of a system of n equations in n variables, where $\underline{0}$ denotes the $n \times 1$ column vector of zeros. If $\det(A) \neq 0$, show that the only solution of the system is the zero solution.

7. (a) For a Boolean algebra $(B, +, \cdot, ', 0, 1)$, show that

(i) $(a + b) + (a' \cdot b') = 1$ for all $a, b \in B$

(ii) $a = 0$ if and only if $b = (a \cdot b') + (a' \cdot b)$ for all $b \in B$.

(b) Consider the following combinatorial circuit:



(i) Find the Boolean function that represents the circuit.

(ii) Simplify the Boolean function.

(iii) Sketch the equivalent simplified circuit.

Continued...,

8. (a) Determine, by using a truth table, whether the proposition

$$(p \rightarrow q) \leftrightarrow (\sim p \vee q)$$

is a tautology.

(b) Stating any result you use, simplify the proposition by using algebra of propositions: $\sim(p \vee q) \wedge (\sim p \wedge (q \Rightarrow \sim p))$.

(c) Determine the validity of the following argument:

"If you attend the class regularly you will pass the course unit. Passing this course unit is necessary for getting the scholarship. Therefore, if you have not got the scholarship, then you have not attended the class regularly."

(d) Prove, by using the method of contrapositive that, for a positive integer k greater than 2, if $2^k - 1$ is a prime then k is an odd integer.

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