University of Kelaniya – Sri Lanka

Centre for Distance and Continuing Education

Bachelor of Science (General) Degree, First Examination (External) - 2012/2013 (New Syllabus)

June/July - 2017

Pure Mathematics – PMAT E1025

Discrete Mathematics I

No. of Questions: **Eight** (08) Number of Pages: **Four** (04) Time: **Three** (03) **Hrs**

Answer **Six** (06) Questions Only.

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01 (a) (i) Let A, B and C be any three subsets of a universal set X. Using the first principles, show that

$$A \cap (B - C) = (A - C) \cap (B - C)$$

(ii) Stating any results you use, show that, for any three subsets A, B and C of a universal set, $(B - A) - C = (A \cup B) - (A \cup C)$.

(iii) Considering the collection $\{A_n : n \in \mathbb{Z}^+\}$ of intervals on the real line where

$$A_n = \left[1, (-1)^n \frac{1}{n}\right].$$

find $\bigcap_{n \in \mathbb{Z}^+} A_n$ and $\bigcup_{n \in \mathbb{Z}^+} A_n$.

(b) (i) Find the truth table of the proposition $(p \wedge q) \Rightarrow (\sim p \vee q)$.

(ii) Stating any results you use, simplify the following proposition

$\sim(p \wedge \sim q) \vee (p \Rightarrow \sim q)$.

02. (a) Consider the following statements.

p: x is less than zero q: x is not positive.

Write down the converse, inverse and contrapositive of the statement " if x is less than zero then x is not positive".

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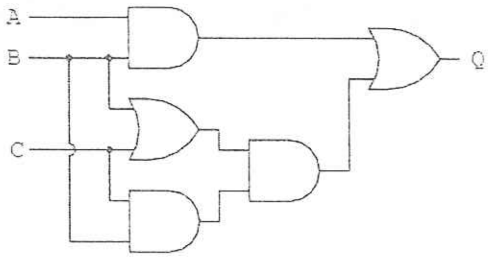
(b) Find a Boolean function of the following combinatorial circuit, simplify it and draw the equivalent circuit.



(c) For the Boolean algebra $[B, +, \cdot, ']$ and $a, b, c \in B$ show that

(i) $a.b' + b.a' = 0 \Leftrightarrow a = b$

(ii) $c.(a+b) + a'.c + b.c' = b + c$

03. (a) (i) Define what is meant by an equivalence relation and an equivalence class.

Show that any two equivalence classes are either equal or disjoint.

(ii) Let $A = \{1, 2, 3, \cdots, 9\}$. R is a relation on $A \times A$ defined by

$(a, b)R(c, d) \Leftrightarrow a + d = b + c$.

Show that R is an equivalence relation and find the equivalence class of $(3, 6)$.

(b) Determine the validity of the following argument.

"If it rains then students do not come to the school. Students came to school. Therefore, it did not rain."

04. (a) Define symmetric and skew symmetric matrices. Show that any square matrix can be written as a sum of a symmetric matrix and a skew symmetric matrix.

(b) Consider the following system of linear equations. For what value of real λ , the Cramer's rule is applicable to solve the linear system

$$\begin{aligned}(1-\lambda)x + 2y + 3z &= 5\\ 3x + (1-\lambda)y + 2z &= 6\\ 2x + 3y + (1-\lambda)z &= -2\end{aligned}$$

Determine the real values of λ for which the system can be solved using Cramer's rule and find the solution using Cramer's rule when λ = 7.

Continued...

05. (a) A function $f: \mathbb{R} \longrightarrow \mathbb{R}$ is defined by

$$f(x) = \begin{cases} \sqrt{x} - 1 : x \geq 1 \\ x - 1 : \quad x < 1 \end{cases}.$$

(i) Sketch the graph of f.

(ii) Show that f is one-to-one and onto.

(iii) Find f^{-1}.

(b) Functions $f: \mathbb{R} \longrightarrow \mathbb{R}$ and $g: \mathbb{R} \longrightarrow \mathbb{R}$ are defined by

$f(x) = x + 1$ and $g(x) = x^2 - 2$.

Find $(g \circ f)^{-1}([-2, -1])$.

06. (a) If a, b and c are nonzero scalars, using the properties of determinants, show that

$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + 1\right).$$

(b) If t is a positive scalar, show that $\begin{vmatrix} a^2+t & ab & ca \\ ab & b^2+t & bc \\ ca & bc & c^2+t \end{vmatrix}$ is nonzero.

07. Consider the following system of equations.

$$\begin{aligned} 2x - 3y + 8z &= 9 \\ 3x + y + \lambda z &= \mu \\ x + 2y - 3z &= 8 \end{aligned}$$

Find the values of λ and μ so that the system of linear equations has

(i) a unique solution

(ii) infinitely many solutions

(ii) no solutions.

Find the solution in case (ii) using parameters.

Continued...

08. (a) Using elementary row operations, find the inverse of the matrix $\begin{pmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{pmatrix}$.

(b) If A and B are two invertible matrices, show that

(i) $(AB)^{-1} = B^{-1}A^{-1}$

(ii) A^T is invertible

(iii) $(A^T)^{-1} = (A^{-1})^T$.

(c) If A is a square matrix such that AA^T is a nonsingular matrix and if $B = A^T(AA^T)^{-1}A$, then show that B is symmetric and $B^2 = B$.