University of Kelaniya – Sri Lanka

Centre for Distance and Continuing Education

Bachelor of Science (General) Degree First Examination – 2012

October/November 2015

Pure Mathematics

PMAT- E 1025 Discrete Mathematics 1

Answer Six (06) questions only

No. of questions : 08 **No. of Pages :** 04 **Time :** 03 **Hours**

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1. (a). Let A, B and C be any three subsets of a universal set E. Show by using the first principles that

   (i) $A \cap (B - C) = (A - C) \cap (B - C)$

   (ii) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

   (iii) $(A \cap B^c) \cup (A \cap B) = A$

   (b). Let $\mathcal{P}(X)$ denote the power set of X. Show that for any two sets A and B

   $$\mathcal{P}(A) \cup \mathcal{P}(B) \subseteq \mathcal{P}(A \cup B).$$

   (c). Let A_n be the open interval on $\mathbb{R}$ defined by $A_n = \left(0\,,\frac{1}{n}\right)$.

   (i) Interpret each of the sets $A_1 \times A_2$, $A_2 \times A_3$ and $A_3 \times A_4$ geometrically.

   (ii) Determine $\bigcap_{n\in\mathbb{Z}^+}(A_n \times A_{n+1})$.

2. Let R be a relation defined on a set A.

   Define

   (i) what is meant by saying that R is an equivalence relation on A and

   (ii) the equivalence class $[x]$ of $x \in A$.

   (a) Let R be a relation defined on $\mathbb{Z}^+ \times \mathbb{Z}^+$ as follows:

   $$(a,b)R(c,d) \text{ if and only if } a + d = b + c$$

   (i) Show that R is an equivalence relation.

   (ii) Find the equivalence class of (1,3).

   (b) Determine whether the following relation R defined on $\mathbb{Z}$ is an equivalence relation:

   $$x\,R\,y \text{ if and only if } x + y \leq 3.$$

Continued...,

3. (a). Show that the following functions are one to one.

(i) $f: \mathbb{R} \longrightarrow \mathbb{R}, f(x) = x^3$,

(ii) $g: \mathbb{R} \longrightarrow \mathbb{R}, g(x) = 2^x + 3$.

(b)

(i) Let $f: A \rightarrow B$ and $g: B \rightarrow C$ be functions such that $g\ o\ f$ is one to one. Show that f is one to one.

(ii). Let $f: \mathbb{R}^+ \rightarrow \mathbb{R}^+$ be the function defined by $f(x) = \frac{1}{x}$. Show that f is one to one and onto, and find the inverse of f.

4. (a) Let A and B are be square matrices of the same order.

(i) Show that $(A + A^T)$ and AA^T are symmetric matrices.

(ii) Show that if A and B are commutative, then A^T and B^T are commutative.

(b) If A and B are two invertible matrices of the same order, then show that

$$adj(AB) = (adjB)(adjA).$$

(c) Let $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$.

Find a formula for A^n $(n \geq 1)$ and verify your formula using mathematical induction.

5. (a) Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix}$.

Show that $A^3 - 4A^2 + 4A - I = 0$ and hence find the inverse of A.

(b) Using the properties of determinants, Show that

$$\begin{vmatrix} 1 & a & a^2 & a^3 + bcd \\ 1 & b & b^2 & b^3 + cda \\ 1 & c & c^2 & c^3 + dab \\ 1 & d & d^2 & d^3 + abc \end{vmatrix} = 0.$$

(c) Let $A = (a_{ij})_{n \times n}$ be the $n \times n$ matrix defined by $a_{ij} = \begin{cases} 3 & if\ i = j \\ 1 & if\ i \neq j \end{cases}$

Show that $\det(A) = 2^{n-1}(n + 2)$.

Continued...,

6. (a) Consider the system of linear equations

$$\begin{aligned} x + ay + a^3z &= a^4 + \lambda a^2 \\ x - by + b^3z &= b^4 + \lambda b^2 \\ x + cy + c^3z &= c^4 + \lambda c^2 \end{aligned}$$

in unknowns x, y and z, where a, b, c are real numbers such that $(a-b)(b-c)(c-a) \neq 0$ and λ is parameter.

Find the values of λ for which the system is consistent when

(i) $a + b + c \neq 0$ and

(ii) $a + b + c = 0$.

(b) If $pqr \neq 0$ and the system of equations

$$\begin{aligned} (pa)x + by + cz &= 0 \\ ax + (q+b)y + cz &= 0 \\ ax + by + (r+c)z &= 0 \end{aligned}$$

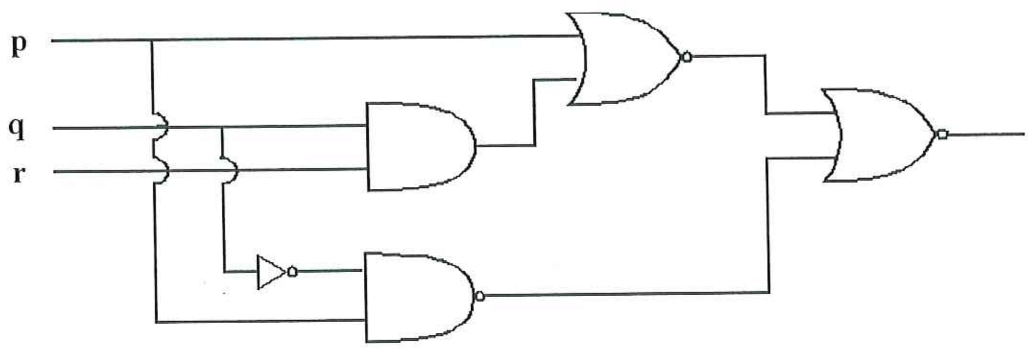
has a non-trivial solution, then show that $\frac{a}{p} + \frac{b}{q} + \frac{c}{r} = -1$.

7. (a) For a Boolean algebra $(B, +, \cdot, ')$ and $a, b, c \in B$, show that

(i) $a \cdot b + a \cdot (b + c) + b \cdot (b + c) = b - a \cdot c$

(ii) $a' \cdot b' \cdot c + a' \cdot b \cdot c + a \cdot b' \cdot c + a \cdot b \cdot c = c$

(c) Consider the following combinatorial circuit:



Continued...,

(i) Find the Boolean function for the circuit.
(ii) Simplify the Boolean function.
(iii) Sketch the equivalent simplified circuit.

8. (a) Find the truth table of the proposition $(p \wedge q) \Longrightarrow (\sim p \vee q)$.

(b) Stating any result you use, simplify the proposition by using algebra of propositions: $\sim(p \wedge \sim q) \wedge (\sim p \wedge q \wedge r) \vee \sim(p \vee \sim q)$.

(c) Determine the validity of the following argument:

"If X and Y are non empty sets then the Cartisian product $X \times Y$ is non-empty. Therefore, if $X \times Y$ is empty, then X and Y are empty sets"

(c) Using the method of contradiction prove that if $a, b \in \mathbb{Z}$, then $a^2 - 4b \neq 2$.

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