# University of Kelaniya - Sri Lanka

## Centre for Open and Distance Learning

Bachelor of Science (General) Degree First Examination (External) – 2007

**Pure Mathematics**
**PMAT E1025-Dicrete Mathematics I**

**No. of Questions : Eight (08)** **No. of Pages : Four (04)** **Time : Three (03) hrs**

Answer <u>Six (06)</u> Questions only.

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1. (a) Let $A$ and $B$ be any two subsets of a universal set $X$.

   (i) Show, by using first principles that $(A \cap B')' = A' \cup B$.

   (ii) Simplify, by using the algebra of sets , the expressions

   (a) $(A-B)\cup(B-A)\cup(A-B')$.

   (b) $[(A'\cap B')-(A\cup B')]\cup A$.

   (b) Let $A_n$ be a sequence of intervals of R defined by $A_n = \left[\frac{1}{n+1}, \frac{1}{n}\right]$.

   Find $\bigcap_{n=1}^{\infty} A_n$ , $\bigcup_{n=1}^{\infty} A_n$ and $\bigcup_{n=1}^{\infty}(A_1 \times A_n)$.

2. (i) Write down the three properties that a relaion $R$ must possess if it is an equivalence relation.

   Suppose that $R$ is the relation on the set of integers defined by $aRb$ if and only if $a+b$ is a multiple of 2. Prove that this is an equivalence relation. What are the corresponding equivalence classes?

   (ii) Prove by contradiction that there is no largest natural number.

3. (i) Suppose that $X, Y, Z$ are sets and $f: X \to Y$ and $g: Y \to Z$.
   Define the composition $g \circ f$ of $f$ and $g$.
   Prove that if $f$ and $g$ are injections, then $g \circ f$ is also an injection.

*Continued...*

(ii) Let $\mathbb{Z}$ denote the set of integers. Explain why it is true that the function $f : \mathbb{Z} \to \mathbb{Z}$ defined by

$$f(x) = \begin{cases} x+1 & ;\text{ if x even} \\ -x+3 & ;\textit{if} \text{ x odd} \end{cases}$$

is a bijection.

What is the inverse function?

4. (a) Using the algebra of prepositions, show that $\sim(p \vee q) \vee (\sim p \wedge q)$ is logically equivalent to $\sim p$.

(b) Simplify the preposition $(\sim p \wedge \sim q) \vee (\sim p \wedge q \wedge r) \vee \sim(p \vee \sim q)$.

(c) Construct a truth table to determine the validity of the following argument: "If *X* and *Y* are non empty sets then the Cartesian product $X \times Y$ is non-empty. Therefore, if $X \times Y$ is empty, then *X* and *Y* are empty sets."

(d) Determine the truth value of the following statement and justify your answer

$$\forall x \in \mathbb{Z} \text{ , } \exists n \in \mathbb{Z} \quad \text{such that} \quad x < n+3 < x^2 - 6x + 13 .$$

5. (i) Find the factors of det *A*, where $A = \begin{pmatrix} a & b & c \\ b & c & a \\ c & a & b \end{pmatrix}$.

Hence find the factors of det *B*, where $B = \begin{pmatrix} 1 & a & a^2 \\ a^2 & 1 & a \\ a & a^2 & 1 \end{pmatrix}$.

(ii) Find the values of $x, y, z, t, u$ and $v$ in terms of $a, b$ and $c$ so that

$$\begin{pmatrix} x+a & t & u \\ b & y+b & v \\ c & a & y+c \end{pmatrix}$$ is a skew-symmetric matrix.

If the matrix so obtained is $A$, without expanding the corresponding determinant, show that $A$ is singular.

*Continued...*

6. (i) The function $f(x)$ is given by

$$f(x)=\frac{a}{1+x^2}+bx+c$$ for some constants $a,b$ and $c$.

Given that $f(0)=8, f(1)=3$ and $f(2)=-8/5$.

Find a system of linear equations in $a,b$ and $c$. Use Cramer's rule to solve this system.

(ii) Consider the system of equations

$$\begin{aligned} x+y+2z &= 1 \\ 2x-y+z &= 2 \\ -x+2y+z &= k \\ y+hz &= 0 \end{aligned}$$

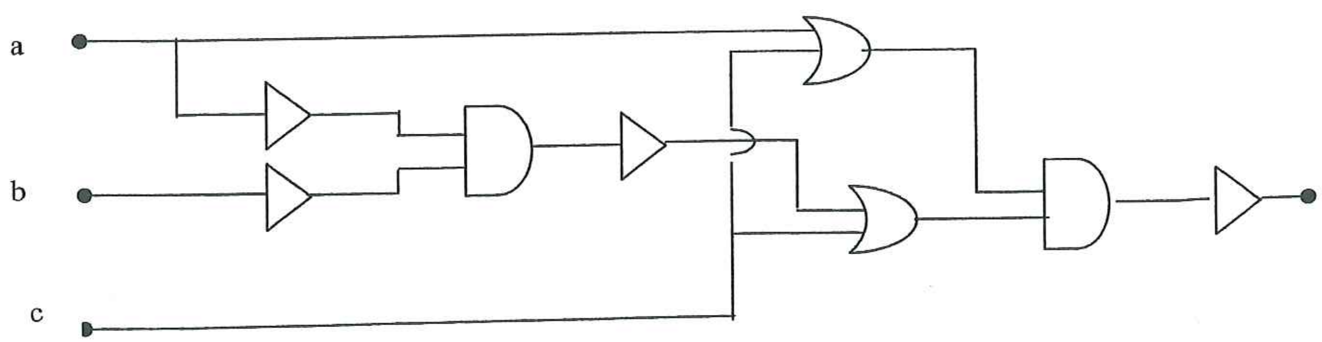
Use Gaussian elimination to find the values of $k$ and $h$ for which this system has

(a) no solution
(b) a unique solution
(c) infinitely many solutions

Find the general solution in case (c).

7. (i) For a Boolean algebra $[B,+,.,']$ and for $a,b,c \in B$, simplify $[(a+b)+(a+b').b]'$.

(ii) Consider the following combinatorial circuit:




(a) Find the Boolean representation for the above circuit.
(b) Simplify the Boolean representation to only three components.
(c) Sketch the equivalent simplified circuit.

*Continued...*

8. (i) Use the mathematical induction to show that
$f(n) = 7^{2n} + 2^{3(n-1)} . 3^{(n-1)}$ is divisible by 25 for all $n \in \mathbb{N}$.

(ii) What is meant by saying the set $S$ has cardinality $m$?

Let $|S|$ denote the cardinality of set $S$.
Let $X$ and $Y$ are two finite sets.
Prove that $|X \cup Y| = |X| + |Y| - |X \cap Y|$.

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