University of Kelaniya – Sri Lanka

Centre for Distance and Continuing Education

Bachelor of Science (General) Degree First Examination – 2012

October/November 2015

Applied Mathematics

AMAT- E 1025

Answer Six (06) questions only

No. of questions : 08 | No. of Pages : 04 | Time : 03 Hours

1. Obtain the radial a nd transverse components of acceleration of a particle, which describes the plane curve $r = f(\theta)$, in the form $\ddot{r} - r\dot{\theta}^2,\ \left(\frac{1}{r}d\left(r^2\dot{\theta}\right)/dt\right)$.

   A particle P of mass m is attached to a rixed point O, by a light inelastic string of length l. The string is taut and P is held at rest with OP making an angle $\alpha < \left[\frac{\pi}{2}\right]$ with the downward vertical. If the particle is given a velocity u horizontally, show that in the usual notation, when, OP, makes an angle θ with the downward vertical,

   $$l^2\left(\dot{\theta}^2 + sin^2\theta\dot{\phi}^2\right) - 2lgcos\theta = u^2 - lgcos\alpha \text{ and } lsin^2\theta\dot{\phi} = usin\alpha.$$

   Hence show that

   $$l^2\dot{\theta}^2 = (cos\alpha - cos\theta)\left[\frac{u^2(cos\alpha + cos\theta)}{sin^2\theta} - 2lg\right].$$

2. (a) Find the moment of inertia of a uniform rod of mass M and length $2a$ about an axis passing through one end perpendicular to the rod.

   (b) A uniform rod AB of length $2a$ and mass m is free to rotate in a vertical plane about the point C on the rod where $AC = x(< a)$. Find the moment of inertia of the rod about C.

   If the rod is released from the rest from a horizontal position taking moment of forces acting on the rod about C, and hence integrating the equation thus obtained, show that, during the subsequent motion,

   $$\dot{\theta}^2 = \frac{6g(a-x)\sin\theta}{a^2 + 3(a-x)^2},$$ where θ is the angle between the rod and the horizontal.

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3. (a) State clearly what is meant by the statement that two systems S_1 and S_2 are equimomental.

(b) State and prove a set of necessary and sufficient conditions for two systems of particles to be equimomental.

(c) $ABCD$ is a uniform parallelogram of mass M. Show that, the parallelogram is equimomental with the system of four masses $\frac{M}{6}$ which are placed at mid points of the four sides and a mass $\frac{M}{3}$ which is placed at the point of intersection of diagonals.

4. (a) In the usual notation show that $\frac{dH}{dt} = \sum_{i=1}^{N} \underline{r_i} \wedge \underline{F_i}$.

(b) Find the moment of inertia of a circular disc of mass M and radius a about an axis passing through its center and perpendicular to the disk.

(c) A circular lamina of weight W and radius a can turn freely about a horizontal axis which passes through a point

O of its circumference and is perpendicular to its plane. The motion commences when the diameter through O is vertically above O.Show that, when the diameter is turned through an angle θ, then components of strain at O along and perpendicular to this diameter are respectively $\frac{W(7cos\theta-3)}{3}$ and $\frac{W(sin\theta)}{3}$.

5. (a) In the usual notation, for a plane lamina, $I_{Ox} = A, I_{Oy} = B$, and $I_{xy} = H$ with respect to two perpendicular axes Ox, Oy in the plane of a lamina.

Obtain the formula $I = A\cos^2\theta - 2H\sin\theta\cos\theta + B\sin^2\theta$ for the moment of inertia I of a lamina about the line $y = x\tan\theta$ lying in the plane of the lamina .

(b) Show that, if the principal axes of above lamina is inclined an angle θ to Ox then $\tan 2\theta = \frac{2H}{B-A}$.

(c) Show that an extremity of the bounding diameter of a uniform semi-circular lamina the principal axis makes an angle $\frac{1}{2}tan^{-1}(\frac{8}{3\pi})$ with the diameter.

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6. (a) Find the moment of inertia of a uniform solid sphere of radius a and mass M about its diameter.

   (b) Show that, in the usual notation, the kinetic energy of a rigid body moving parallel to a fixed plane, is given by

$$T = \frac{1}{2}MV_G^2 + \frac{1}{2}I\omega^2.$$

   (c) Velocity of the center of a uniform solid sphere of mass M and radius a rolling without sliding along a straight line on a smooth horizontal table is given by v.

   Show that the total kinetic energy of the sphere is $\left(\frac{7}{10}\right)Mv^2$.

7. (a) The direction cosine of a straight line with respect to a Cartesian coordinate system with the origin O, is given by $[l, m, n]$. In the usual notation, using moments of inertia A, B, C and products of inertia E, F, G, show that the moment I_l of a mass distribution About the above straight line is given by the following equation,

   $I_l = Al^2 + Bm^2 + Cn^2 - 2Fmn - 2Gln - 2Hlm.$

   (b) Show that the momental ellipsoid at a point of the rim of a hemisphere is

$$2x^2 + 7(y^2 + z^2) - \left(\frac{15}{4}\right)xz = constant.$$

8. (a) Define the angular momentum $\underline{H}_0$ of a system of particles about a point O .

   (b) Show that, in the usual notation, for a rigid body rotating with an angular velocity $\underline{\omega}$ about an axis through O , $\underline{H}_0$ can be written as $\underline{H}_0 = \sum m_i[r_i^2\underline{\omega} - (\underline{r}_i.\underline{\omega})\underline{r}_i]$.
   Show also that, in the usual notation, the components H_x, H_y, H_z of $\underline{H}_0$ along three rectangular Cartesian axes at O can be expressed as

$$\begin{pmatrix} H_x \\ H_y \\ H_z \end{pmatrix} = \begin{pmatrix} A & -F & -E \\ -F & B & -D \\ -E & -D & C \end{pmatrix} \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix}.$$

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