



University of Kelaniya – Sri Lanka
Centre for Distance and Continuing Education
Bachelor of Science (General) Degree First Examination – 2012
October/November 2015
Applied Mathematics
AMAT- E 1025
Answer Six (06) questions only

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Time : 03 Hours

1. Obtain the radial and transverse components of acceleration of a particle, which describes the plane curve $r = f(\theta)$, in the form $\ddot{r} - r\dot{\theta}^2$, $\left(\frac{1}{r}d(r^2\dot{\theta})/dt\right)$.

A particle P of mass m is attached to a fixed point O , by a light inelastic string of length l . The string is taut and P is held at rest with OP making an angle $\alpha < \left[\frac{\pi}{2}\right]$ with the downward vertical. If the particle is given a velocity u horizontally, show that in the usual notation, when, OP , makes an angle θ with the downward vertical,

$$l^2(\dot{\theta}^2 + \sin^2\theta\dot{\phi}^2) - 2l g \cos\theta = u^2 - l g \cos\alpha \text{ and } l \sin^2\theta\dot{\phi} = u \sin\alpha.$$

Hence show that

$$l^2\dot{\theta}^2 = (\cos\alpha - \cos\theta) \left[\frac{u^2(\cos\alpha + \cos\theta)}{\sin^2\theta} - 2lg \right].$$

2. (a) Find the moment of inertia of a uniform rod of mass M and length $2a$ about an axis passing through one end perpendicular to the rod.

- (b) A uniform rod AB of length $2a$ and mass m is free to rotate in a vertical plane about the point C on the rod where $AC = x (< a)$. Find the moment of inertia of the rod about C .

If the rod is released from the rest from a horizontal position taking moment of forces acting on the rod about C , and hence integrating the equation thus obtained, show that, during the subsequent motion,

$$\dot{\theta}^2 = \frac{6g(a-x)\sin\theta}{a^2 + 3(a-x)^2}, \text{ where } \theta \text{ is the angle between the rod and the horizontal.}$$

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3. (a) State clearly what is meant by the statement that two systems S_1 and S_2 are equimomental.
- (b) State and prove a set of necessary and sufficient conditions for two systems of particles to be equimomental.
- (c) $ABCD$ is a uniform parallelogram of mass M . Show that, the parallelogram is equimomental with the system of four masses $\frac{M}{6}$ which are placed at mid points of the four sides and a mass $\frac{M}{3}$ which is placed at the point of intersection of diagonals.
4. (a) In the usual notation show that $\frac{dH}{dt} = \sum_{i=1}^N \underline{r}_i \wedge \underline{F}_i$.
- (b) Find the moment of inertia of a circular disc of mass M and radius a about an axis passing through its center and perpendicular to the disk.
- (c) A circular lamina of weight W and radius a can turn freely about a horizontal axis which passes through a point O of its circumference and is perpendicular to its plane. The motion commences when the diameter through O is vertically above O . Show that, when the diameter is turned through an angle θ , then components of strain at O along and perpendicular to this diameter are respectively $\frac{W(7\cos\theta-3)}{3}$ and $\frac{W(\sin\theta)}{3}$.
5. (a) In the usual notation, for a plane lamina, $I_{Ox} = A, I_{Oy} = B$, and $I_{xy} = H$ with respect to two perpendicular axes Ox, Oy in the plane of a lamina.
- Obtain the formula $I = A\cos^2\theta - 2H\sin\theta\cos\theta + B\sin^2\theta$ for the moment of inertia I of a lamina about the line $y = x\tan\theta$ lying in the plane of the lamina.
- (b) Show that, if the principal axes of above lamina is inclined an angle θ to Ox then $\tan 2\theta = \frac{2H}{B-A}$.
- (c) Show that an extremity of the bounding diameter of a uniform semi-circular lamina the principal axis makes an angle $\frac{1}{2}\tan^{-1}\left(\frac{8}{3\pi}\right)$ with the diameter.

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6. (a) Find the moment of inertia of a uniform solid sphere of radius a and mass M about its diameter.
- (b) Show that, in the usual notation, the kinetic energy of a rigid body moving parallel to a fixed plane, is given by

$$T = \frac{1}{2}MV_G^2 + \frac{1}{2}I\omega^2.$$

- (c) Velocity of the center of a uniform solid sphere of mass M and radius a rolling without sliding along a straight line on a smooth horizontal table is given by v .

Show that the total kinetic energy of the sphere is $\left(\frac{7}{10}\right)Mv^2$.

7. (a) The direction cosine of a straight line with respect to a Cartesian coordinate system with the origin O , is given by $[l, m, n]$. In the usual notation, using moments of inertia A, B, C and products of inertia E, F, G , show that the moment I_l of a mass distribution About the above straight line is given by the following equation,

$$I_l = Al^2 + Bm^2 + Cn^2 - 2Fmn - 2Gln - 2Hlm.$$

- (b) Show that the momental ellipsoid at a point of the rim of a hemisphere is

$$2x^2 + 7(y^2 + z^2) - \left(\frac{15}{4}\right)xz = \text{constant}.$$

8. (a) Define the angular momentum \underline{H}_O of a system of particles about a point O .
- (b) Show that, in the usual notation, for a rigid body rotating with an angular velocity $\underline{\omega}$ about an axis through O , \underline{H}_O can be written as $\underline{H}_O = \sum m_i[r_i^2 \underline{\omega} - (\underline{r}_i \cdot \underline{\omega})\underline{r}_i]$. Show also that, in the usual notation, the components H_x, H_y, H_z of \underline{H}_O along three rectangular Cartesian axes at O can be expressed as

$$\begin{pmatrix} H_x \\ H_y \\ H_z \end{pmatrix} = \begin{pmatrix} A & -F & -E \\ -F & B & -D \\ -E & -D & C \end{pmatrix} \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix}.$$

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