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University of Kelaniya-Sri Lanka

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External Examinations Branch

විද්‍යා පීඨය - Faculty of Science

විද්‍යාවේදී (සාමාන්‍ය) උපාධි ප්‍රථම පරීක්ෂණය (බාහිර) - 2008

2010 ඔක්තෝබර්

Bachelor of Science (General) Degree First Examination (External) 2008

October -2010

Applied Mathematics - AMAT E 1025

No. of Questions : Eight (08) **No. of Pages : Four (04)** **Time : Three (03) hrs**

Answer Six (06) Questions only.

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1. Explain briefly what is meant by an inertial frame.

(i) A small spherical metal smooth ball is free to slip on the horizontal smooth surface of a table. Initially, the ball is at rest and the table starts to move horizontally with uniform acceleration *f*, Describes the motion of the ball, and hence show that a reference frame fixed to the table can not be an inertial frame.

(ii) Using the coordinate transformation equations

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}, \quad y' = y, \quad z' = z$$

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}},$$ in the usual notation, find the relative velocity between two particles moving along the same straight line with uniform velocities. Show that the relative velocity is equal to the well known velocity difference if the velocities concerned are very small compared to the speed of light.

Continued...

2. A uniform wheel with a diametrical spoke is free to move in a vertical plane about the horizontal axes through the centre O, which is fixed. The wheel starts to rotate with uniform angular velocity ω in the clock-wise sense and a smooth small bead of mass m is free to slide along the spoke. If the radius of the wheel is a, and initially the beads is at rest at the top of the wheel, show that,

$$r = Ae^{\omega t} + Be^{-\omega t} + \frac{g}{2\omega^2} \cos \omega t$$

when the bead is at a distance r from the centre O at time t, where constants A, B are to be determined.

Find the reaction of the spoke on the bead.

3. Show that orbit of a body moving under a central force P per unit mass is given by $\frac{d^2u}{d\theta^2} + u = \frac{P}{h^2u^2}$ in the usual notation.

A particle, moving under the central force $2\mu(u^3 - a^2u^5)$ per unit mass, is projected with velocity $\frac{\sqrt{\mu}}{a}$ along an apsis at a distance a from the origin. If the particle is at a distance r from the origin at time t, show that t is given by

$$\sqrt{\mu}\, t = \int_a^r \frac{r^2\, dr}{\sqrt{r^2 - a^2}}.$$

4. A particle is projected with velocity $\underline{u}$ from a point of earth surface which is taken as the origin of a reference frame. Neglecting air resistance, local variation of gravity and term of order ω^2, show in the usual notation that its position vector $\underline{r}$ at time t, is given by

$$\underline{r} = \underline{u}t + \frac{1}{2}\underline{g}t^2 - (\underline{\omega} \times \underline{u})\, t^2 - \frac{1}{3}(\underline{\omega} \times \underline{g})\, t^3$$

A shell is fired horizontally due south from the top of a cliff of height h over the sea, at co-latitude θ.

Continued...

Neglecting air resistance and terms of order higher than one of ω, show that in order that the shell may hit the sea due south-east of its point of firing, its initial velocity u must be given by

$$u = \frac{2h\,\omega \sin\theta}{3[1+\left(\frac{2h}{g}\right)^{\frac{1}{2}}\omega \cos\theta]}$$

[You may assume the equation $\ddot{\underline{r}} - 2\underline{\omega}\times\dot{\underline{r}} = \underline{g}$ in the usual notation].

5. Define the "principal axes" at a point of rigid body.

Moments and product of inertia of a plane lamina with respect to ox, oy lying in the plane of the lamina are A, B and H respectively. Show that the moment of inertia of the lamina about an axis through O lying in the plane of the lamina and making an angle θ with ox is

$$A\cos^2\theta + B\sin^2\theta - 2H\sin\theta\cos\theta .$$

Show also that principal axes, lying in the plane of the lamina at O, are inclined at angle θ and $\theta+\frac{\pi}{2}$ with ox, where $\tan 2\theta = \frac{2H}{B-A}$.

$ABCDEF$ is a regular hexagon of a side $4a$, mass 36m and centre O. If the portion $OBCD$ out off from the lamina, find the principal axes of $ABODEF$ at F.

[You may assume that a uniform triangular lamina of mass $3m$ is equimomental with three particle each of mass m placed at the mid points of the sides of the lamina.]

6. Obtain, in the usual notation, the expression

$$\frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 .$$

for the kinetic energy of a lamina moving in a plane.

Continued...

A uniform rod of mass m and length $2a$ is held inclined to the vertical at an angle α with its lower end in contact with a smooth horizontal table and is released from rest. When the inclination of the rod to the vertical is θ, prove that the reaction of the table is

$$\frac{mg[4-6\cos\alpha\cos\theta+3\cos^2\theta]}{\left(1+3\sin^2\theta\right)^2}.$$

Prove also that the rod never leaves the table.

7. A uniform spherical shell is projected up a line of greatest slope on a rough plane inclined at an angle α, to the horizontal with velocity u and back spin Ω. If $u > 4a\Omega$ and the coefficient of friction between the sphere and the plane is $\frac{1}{5}\tan\alpha$, show that the motion is at first slipping and it will stop slipping before it will stop moving up the plane. Show that the sphere moves up for a time $\frac{5u-4a\Omega}{12g\sin\alpha}$. Show also that the sphere does not roll throughout its motion.

8. For a rigid body moving about a fixed point O of it, show in the usual notation that

$$T=\frac{1}{2}\left[A\omega_1^2+B\omega_2^2+C\omega_3^2\right]$$

$$H^2=A^2\omega_1^2+B^2\omega_2^2+C^2\omega_3^2$$

where A, B, C are the principal moments about O.

If $A<B<C$ and $H^2=2BT$, show that

$$\omega_2=\frac{q\left(Fe^{2q\mu t}-1\right)}{Fe^{2q\mu t}+1}$$

where $q^2=\frac{2T}{B}$, $\mu=\sqrt{\frac{(B-A)(C-B)}{AC}}$ and F is a constant.

[Assume that the moment of external force about O is zero].

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