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University of Kelaniya-Sri Lanka

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External Examinations Branch

විද්‍යා පීඨය - **Faculty of Science**

විද්‍යාවේදී (සාමාන්‍ය) උපාධි ප්‍රථම පරීක්ෂණය (බාහිර) - 2008 හා 2009

2010 ඔක්තෝබර්

Bachelor of Science (General) Degree First Examination (External) 2008 & 2009

October -2010

APPLIED MATHEMATICS

AMAT E1015 - Vector Algebra and Vector Analysis

No. of Questions : Eight (08) **No. of Pages : Three (03)** **Time : Three (03) hrs**

Answer <u>Six (06)</u> Questions only.

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01. Let the coordinates of four points A, B, C and D are $(-6,1,6)$, $(6,-2,3)$, $(-2,-3,-1)$ and $(-5,-9,-7)$ respectively, with respect to mutually perpendicular axes.

   i. Show that the points A, B and D lie on the surface of a sphere with centre C.
   ii. Find the coordinates of the point E such that a diameter of the sphere is DCE.
   iii. Show that $B\hat{C}A$ is a right angle and $B\hat{C}E = E\hat{C}A$.
   iv. Show that the diameter DCE does not lie on the plane of the triangle ABC.

02. Let A, B, C and D are four points with rectangular Cartesian coordinates $(0,1,2)$, $(3,0,1)$, $(4,3,6)$ and $(2,3,2)$ respectively.
   Find,
   i. the area of the triangle ABC,
   ii. the perpendicular distance from the point A to the line BC,
   iii. the volume of the tetrahedron $ABCD$,
   iv. the perpendicular distance from the point D to the plane ABC,
   v. the shortest distance between the lines AB and CD.

03. a. Find an equation of the plane containing the points, $P_1(2,-1,1)$, $P_2(3,2,-1)$, and $P_3(-1,3,2)$.

   b. Given that $3\underline{i} + \underline{j} + 2\underline{k}$ and $\underline{i} - 2\underline{j} - 4\underline{k}$ are the position vectors of the points P and Q respectively.

   i. Find an equation of the plane passing through Q and perpendicular to the line PQ.
   ii. What is the distance to the plane above from the point $(-1,1,1)$?

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04. a. A particle moves along the curve $x = t^3 + 1$, $y = t^2$, $z = 2t + 5$,where t is time . Find the components of velocity and acceleration of the particle in the direction $\underline{i} + \underline{j} + 3\underline{k}$ at time $t = 1$.

b. If $\underline{r} = \underline{a}\cos(\omega t) + \underline{b}\sin(\omega t)$, show that

i. $\underline{r} \times \dfrac{d\underline{r}}{dt} = \omega(\underline{a} \times \underline{b})$,

ii. $\dfrac{d^2\underline{r}}{dt^2} = -\omega^2 \underline{r}$,

where $\underline{a}$, $\underline{b}$, ω being constants.

05. a. Sketch the space curve $x = 3\cos t$, $y = 3\sin t$, $z = 4t$ and find the followings .

i. the unit tangent T ,
ii. the principal normal N , curvature κ and radius of curvature ρ ,
iii. the binormal B , torsion τ and radius of torsion σ .

b. Show that the radius of curvature of the curve with parametric equations $x = x(s)$, $y = y(s)$, $z = z(s)$ is given by

$$\rho = \left[\left(\frac{d^2x}{ds^2}\right)^2 + \left(\frac{d^2y}{ds^2}\right)^2 + \left(\frac{d^2z}{ds^2}\right)^2\right]^{-\frac{1}{2}} .$$

06. a. i. Prove that $grad\, f(\underline{r}) \times \underline{r} = \underline{0}$.

ii. If $f = (\underline{a} \times \underline{r})r^n$ show that $div\, f = 0$ and $curl\, f = (n+2)r^n \underline{a} - nr^{n-2}(\underline{a}.\underline{r})\underline{r}$.

b. A vector $\underline{V}$ is called irrotational if $\underline{\nabla} \times \underline{V} = \underline{0}$.
Find constants a, b, c so that
$\underline{V} = (x + 2y + az)\underline{i} + (bx - 3y - z)\underline{j} + (4x + cy + 2z)\underline{k}$ is irrotational .

Continued...

07. a. Show that

i. $grad(\phi\psi) = \phi\, grad\psi + \psi\, grad\phi$,

ii. $div(\phi\, \underline{u}) = \phi\, div(\underline{u}) + grad\,(\phi).\underline{u}$,

iii. $curl(\phi\, \underline{u}) = \phi\, curl(\underline{u}) + grad(\phi) \times \underline{u}$.

b. Let $\underline{a}$ be a constant vector, $\underline{r}$ be the position vector of a point (x, y, z) with respect to a origin O and $r = |\underline{r}|$.

Show that

i. $\underline{\nabla} . \dfrac{\underline{r}}{r^3} = \underline{0}$,

ii. $curl \dfrac{\underline{a} \times \underline{r}}{r^3} = -\dfrac{\underline{a}}{r^3} + \dfrac{3\underline{r}}{r^5}(\underline{a}.\underline{r})$,

iii. $grad\left(\dfrac{\underline{a}.\underline{r}}{r^3}\right) + curl\left(\dfrac{\underline{a} \times \underline{r}}{r^3}\right) = \underline{0}$.

08. a. Write down the divergence theorem .

Show that $\int_S (ax\underline{i} + by\underline{j} + cz\underline{k}).d\underline{a} = \dfrac{4}{3}\pi(a+b+c)$, where S is the surface of the sphere $x^2 + y^2 + z^2 = 1$.

b. If $\underline{F}$ is normal to each point on the surface S, show that $\int_V curl\underline{F}.dV = \underline{0}$.

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