University of Kelaniya- Sri Lanka
Center for Distance & Continuing Education
Bachelor of Science (General) External Degree Examination - 2026 March
Academic Year 2024 - Semester II

STAT 17533 - Probability Distributions & Applications I

No of Questions: Four (04) No. of pages: Two (02) Time: Two & half ($2\frac{1}{2}$) Hours

Answer All Questions

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1. (a) If A and B are independent events, prove that the following pairs of events are also independent:
   (i) A' and B
   (ii) A' and B'

   (b) A batch of 500 containers of frozen orange juice contains 5 defective containers. Two are selected at random without replacement from the batch.
   (i) What is the probability that the second one selected is defective, given that the first one was defective?
   (ii) What is the probability that both are defective?
   (iii) What is the probability that both are acceptable?

   (c) Sixty percent of the students at a certain school wear neither a ring nor a necklace. Twenty percent wear a ring, and thirty percent wear a necklace. If one of the student is chosen at random, what is the probability that this student is wearing
   (i) a ring or a necklace;
   (ii) a ring and a necklace?

2. (a) The probability density function (pdf) of a random variable X is given by

$$f_X(x) = \begin{cases} c(x-1)^2, & 0 < x < 2 \\ 0, & \text{elsewhere,} \end{cases}$$

   (i) Find the value of c such that $f(x)$ is a legitimate pdf.
   (ii) Derive the Cumulative Distribution Function (CDF) of X.
   (iii) Calculate $P(X > 1)$.
   (iv) Calculate $E(X)$.
   (v) Find $P(X > \frac{3}{2} | X > 1)$.

(b) A company producing cereals offers a toy in every sixth cereal package in celebration of their 50th anniversary. A father immediately buys 20 cereal packages.

(i) What is the probability of finding 4 toys in the 20 packages?

(ii) Suppose that the 20 packages the father bought contain three toys. What is the probability that among the 5 packages that are given to the family's youngest daughter, she finds two toys?

3. (a) If the random variable X has the continuous uniform distribution within the interval (a, b), show that the moment generating function (MGF) of X is

$$M_X(t) = \frac{e^{bt} - e^{at}}{t(b-a)}.$$

(b) Let $X \sim Poisson(\lambda)$ where $P(X = x) = \dfrac{\lambda^x e^{-\lambda}}{x!}$ for $x = 0, 1, 2, \ldots$. Derive the probability generating function (PGF) of X.

(c) During a laboratory experiment, the average number of radioactive particles passing through a counter in 1 millisecond is 4. What is the probability that 6 particles enter the counter in a given millisecond?

4. (a) Let the random variable X has an exponential distribution with parameter λ. Find the probability density function of $Y = X^2$.

(b) The waiting time (X in minutes) at a customer service center is uniform distribution within the interval $(0, 25)$.

(i) Find a lower bound for the probability of X within two standard deviations of mean. State the inequality you used to obtain the answer.

(ii) Calculate the exact probability for part (i).

(c) A company who produces tea pack of 500g has the policy of accepting tea packs within the weight range (455g, 545g). If the weights of tea packs in a certain batch were normally distributed with mean 500.2g and standard deviation 14.5g, what is the rejection probability of a randomly picked pack of this batch?

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