



University of Kelaniya – Sri Lanka
Centre for Distance & Continuing Education
Bachelor of Science (General) External
First year Second semester examination(Repeat) - 209 (2024 August)
(New Syllabus)
Faculty of Science
Statistics
STAT 17533 (R) – Probability Distributions & Applications I

No. of Questions: four (04) No. of Pages: three (03) Time: **Two & half (2 1/2) Hours.**

Answer all questions.

1. (a) A random variable X follows a Poisson distribution with rate parameter λ . Write down the expressions for the mean, variance and standard deviation of X .

- (b) The probability distribution of weekly air conditioning units ordered by the “Cooling Partner” company is given below:

Number of air conditioners (X)	1	2	3
$P[X = x]$	1/4	1/2	1/4

If the retailer gains a profit of Rs 10000 from a unit sold, find the mean weekly profit.

- (c) Suppose that the average number of customer complaints received daily at a call center equals 5. Calculate the probability that
- no complaints,
 - at least one complaint,
 - not more than two complaints occur on a given day.
 - What is the probability that there will be at least one complaint in a given week?

2. (a) A discrete random variable X has a binomial distribution with probability mass function given by

$$P[X = x] = \begin{cases} \binom{n}{x} p^x (1-p)^{n-x}; & \text{for } x = 1, 2, \dots, n \\ 0 & ; \text{otherwise} \end{cases}$$

- (i) Prove that the mean of X is equal to np .
(ii) Write down an expression for the variance of X in terms of n and p .
- (b) A factory produces light bulbs, and it is known that 2% of the bulbs are defective. A quality control inspector randomly selects 20 bulbs from the production line.
- (i) What is the probability that exactly 2 of the 20 bulbs are defective?
(ii) What is the probability that no more than 1 of the 20 bulbs is defective?
(iii) What is the probability that at least 3 of the 20 bulbs are defective?
(iv) Calculate the mean and standard deviation of the number of defective bulbs in a sample of 20.
3. (a) Write down the properties of a probability density function.
- (b) Suppose that the lifetime in hours of a certain type of battery, X , is a continuous random variable with probability density function,

$$f(x) = \begin{cases} \frac{k}{x^3}; & x \geq 100 \\ 0; & \text{otherwise.} \end{cases}$$

- (i) Show that $k = 20000$.
(ii) What is the probability that a battery will last less than 150 hours?
(iii) What is the probability that a battery will last more than 200 hours?
(iv) What is the probability that a battery will last less than 150 hours, if it is known that the battery is still functioning after 120 hours of service?
(v) Find the expected lifetime of a battery.
(vi) A random sample of five batteries are selected and tested. What is the probability that exactly one of the four batteries will fail during the first 150 hours of operation?

4. (a) The weight of the packets of a particular snack produced by a machine is normally distributed with a mean of 50 grams and a standard deviation of 0.5 grams.
- (i) What proportion of packets weigh less than 49.5 grams?
 - (ii) What percentage of packets weigh more than 51 grams?
 - (iii) Compute the 95th percentile of packet weights.
 - (iv) If 8 packets of snack is chosen randomly, find the probability that all packets weigh less than 49.5 grams.

- (b) The cumulative distribution function of a discrete random variable Y is given by

$$F(y) = \begin{cases} 0, & y < 0 \\ \frac{3}{28}, & 0 \leq y < 1 \\ \frac{9}{14}, & 1 \leq y < 2 \\ 1, & y \geq 2. \end{cases}$$

- (i) Derive the probability mass function.
- (ii) Find the mean and the variance of the random variable.
- (iii) Find the moment generating function (MGF) of Y .

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A normal distribution curve is shown. The area under the curve to the left of a point labeled z on the horizontal axis is shaded with diagonal lines.

A normal distribution curve is shown. The area under the curve to the left of a point labeled z on the horizontal axis is shaded with diagonal lines.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998