University of Kelaniya – Sri Lanka
Centre for Distance & Continuing Education
Bachelor of Science (General) External
First year Second semester examination - 2019 (2023 March)
(New Syllabus)
Faculty of Science

Statistics
STAT 17533 – Probability Distributions & Applications I

No. of Questions: five (05) No. of Pages: three (03) Time: **Two & half (2 1/2) Hours.**

Answer **Four (04)** questions only.

---

1. *(a)* Let X be a Poisson distributed random variable with probability mass function

$$P[X = x] = \frac{e^{-\lambda}\lambda^x}{x!} \; ; x = 0,1,2,\ldots$$

Prove that $E(X) = \lambda$.

(b) The probability distribution of weekly air conditioning units ordered by the "Cooling Partner" company is given below:

Number of air conditioners (X)	0	5	10
$P[X = x]$	1/4	1/2	1/4

If the retailer gains a profit of Rs 2500 from a unit sold, find the mean weekly profit.

(c) Suppose that the average number of accidents occurring weekly on a particular stretch of a highway equals 3.

(i) Calculate the probability that at least one accident occur in a given week.

(ii) What is the probability that there will be at least one accident in a given month?

2. *(a)* A random variable X has a Binomial distribution with parameters n and p with probability mass function

$$P[X = x] = \binom{n}{x} p^x (1-p)^{n-x} \quad x = 0,1, \dots, n.$$

Suppose that for sufficiently small p and sufficiently large n, such that $np = \lambda$ where λ is a constant. Show that the above Binomial probability can be approximated by a Poisson probability with parameter λ.

(b) When Stephan plays chess against a computer program, he wins with probability 0.6 and loses with probability 0.1 and 30% of the games result in a draw.

(i) Find the probability that Stephan wins 7 games out of 10 games he played.

(ii) Find the probability that Stephan's first win happens when he played his 3^{rd} game.

(iii) Find the probability that Stephan's fifth win happens when plays his eighth game.

(c) Mass produced needles are packed in boxes of 1000. It is believed that 1 needle in 2000 on average is defective.

(i) What is the exact probability that a box contains no defects.

(ii) Stephan says he can accurately approximate the above probability using Poisson approximation. Do you agree with Stephan? Give reasons.

3. *(a)* Write down the properties of a probability density function.

(b) Suppose that the life in hours of a radio tube is a continuous random variable (X). Its probability density function as follows:

$$f(x) = \begin{cases} \dfrac{k}{x^2}; & x \geq 100 \\ 0; & otherwise \end{cases}$$

(i) Show that $k = 100$.

(ii) What is the probability that a tube will last less than 150 hours?

(iii) What is the probability that a tube will last less than 150 hours, if it is known that the tube is still functioning after 50 hours of service?

(iv) Find the expected lifetime of a radio tube.

(v) A random sample of three radio tubes are selected and tested. What is the probability that all three radio tubes will fail during the first 150 hours of operation?

(vi) Find the cumulative distribution function for the lifetime of a radio tube.

4. *(a)* The fill volume of cans filled by a certain machine is normally distributed with mean 12.05 oz and standard deviation of 0.03 oz.

   (i) What proportion of cans contains less than 12 oz.

   (ii) Compute the 99th percentile of fill volumes.

   (iii) If you randomly selected 6 cans, find the probability that at least two cams contain less than 12 oz.

   (b) The cumulative distribution function of a random variable Y is given by

$$F(y) = \begin{cases} 0, & y < 0 \\ \frac{3}{28}, & 0 \le y < 1 \\ \frac{9}{14}, & 1 \le y < 2 \\ 1, & y \ge 2. \end{cases}$$

   (i) Derive the probability density function.

   (ii) Find the mean and the variance of the random variable.

   (iii) What is the moment generating function (MGF) of Y.

   (iv) Find the probability generating function of Y.

5. The probability density function of a Gamma distribution is given by

$$f(x) = \frac{1}{\Gamma(\alpha)\theta^{\alpha}} x^{\alpha-1} e^{-\frac{x}{\theta}} \quad if \ \ 0 \le x \le \infty$$

where $\alpha > 0$ and $\theta > 0$.

(a) Show that the Moment Generating Function (MGF) of the Gamma distribution is given by

$$M(t) = \left(\frac{1}{1-\theta t}\right)^{\alpha} \quad if \ \ t < \frac{1}{\theta}$$

(b) Find the mean and the variance of the Gamma distribution.

(c) Suppose that an average of 30 customers per hour arrive at a gift shop. Let X denotes the waiting time in minutes until the second customer arrives. Assume that X has a Gamma distribution with $\alpha = \theta = 2$.

   (i) What is the probability that the shopkeeper will wait more than 5 minutes before both of the first two customers arrive?

   (ii) Compute the mean and the standard deviation of X. Interpret your answers.

----------/End of Paper/----------