



University of Kelaniya- Sri Lanka
Faculty of Science
Centre for Distance & Continuing Education
Bachelor of Science (General) Degree
First Examination-External
April 2026
Academic Year 2024- Semester II

PURE MATHEMATICS | PMAT 17532 - Discrete Mathematics II

No. of Questions: Five (05) No. of Pages: Four (04) Time: Two (02) hours

Answer only FOUR (04) questions.

1. (a) i. Let $a, b, c \in \mathbb{Z}$ and $a \mid c$. Show that

$$a \mid (b + c) \text{ if and only if } a \mid b.$$

15 Points.

- ii. Let $m, n \in \mathbb{Z}$. Prove that if $2 \mid (m^2 + n^2)$, then m and n are both even or both odd.

15 Points.

- (b) i. Prove that the greatest common divisor of two integers a and b , not both zero, is the least positive integer d such that $d = ma + nb$, for some integers m and n .

15 Points.

- ii. By stating any results you use, prove that for any $a, b \in \mathbb{Z}$, not both zero,

$$\gcd(3a - 2b, 5a - 4b) \mid a.$$

15 Points.

- iii. Find the greatest common divisor d of 910 and 374, and write d is in the form $d = am + bn$, where $m, n \in \mathbb{Z}$.

20 Points.

- (c) State the Fundamental Theorem of Arithmetic.

05 Points.

Use the Fundamental Theorem of Arithmetic to prove that if n^2 is divisible by 12, then n is divisible by 6

15 Points.

Total: 100 Points.

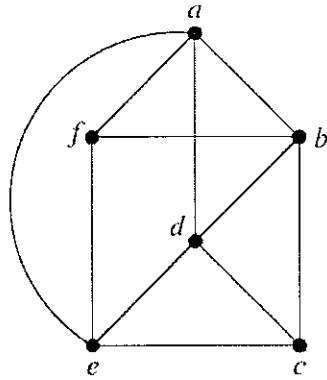
2. (a) Let m be a positive integer. Define what is meant by saying that a is congruent to b modulo m .

05 Points.

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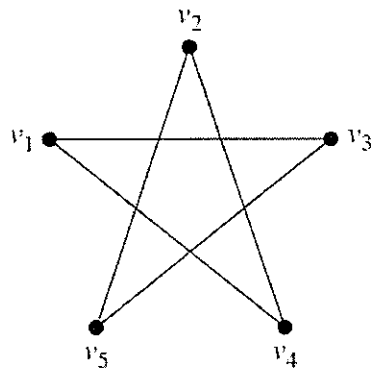
- i. Prove that if $a \equiv b \pmod{m}$, then $2a \equiv 2b \pmod{m}$.
Is the converse true? That is, does $2a \equiv 2b \pmod{m}$ imply $a \equiv b \pmod{m}$?
Explain.
- 25 Points.**
- ii. Find the inverse of 3 modulo 11.
- 15 Points.**
- iii. Find the remainder when 4^{49} is divided by 62.
- 20 Points.**
- (b) i. If s is a solution to $ax \equiv b \pmod{m}$, prove that $a(s + km) \equiv b \pmod{m}$ for any integer k .
- 10 Points.**
- ii. Find all incongruent solutions of x satisfying the equation $24x \equiv 16 \pmod{40}$.
- 25 Points.**
- Total: 100 Points.**
3. (a) State the condition on a, b and c such that the equation $ax + by = c$ has integer solutions.
- 05 Points.**
- For each of the following Diophantine equation, find the number of positive integer solutions:
- i. $8x + 12y = 40$
- 25 Points.**
- ii. $4x + 6y = 5$
- 10 Points.**
- (b) i. How many bit strings of length 8, start with 1 and end with 0?
- 10 Points.**
- ii. How many numbers must be selected from the set $\{1, 3, 5, 7, 9, 11, 13, 15\}$ to guarantee that at least one pair of these numbers add up to 16?
- 20 Points.**
- (c) i. State the **binomial theorem**.
- 10 Points.**
- ii. Using the **binomial theorem** find the coefficient of x^8y^9 in the expansion of $(3x + 2y)^{17}$.
- 20 Points.**
- Total: 100 Points.**
4. (a) i. State the **Handshaking theorem**.
- 10 Points.**
- ii. Let G be a simple graph with 7 edges. If the complementary graph, \overline{G} has 8 edges, how many vertices does G have?
- 30 Points.**
- Continued...**

(b) Consider the following graph.



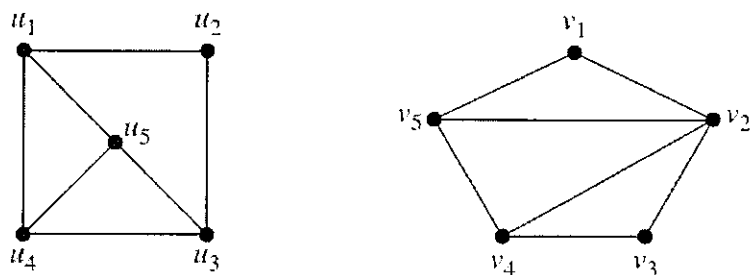
- i. Determine whether graph contain an Euler cycle, an Euler path or neither. Justify your answer. 15 Points.
 - ii. Does the graph contain a Hamilton cycle? Justify your answer. 10 Points.
 - iii. Draw the subgraph induced by the vertex set $\{a, b, d, e\}$. 05 Points.
 - iv. Find the chromatic number of the graph. 20 Points.
 - v. Determine whether the given graph is bipartite. Justify your answer. 10 Points.
- Total: 100 Points.

5. (a) i. Write down the corresponding adjacency matrix of the cycle graph C_5 and show that C_5 is isomorphic to the following graph.



30 Points.
Continued...

- ii. Determine whether the following pair of graphs is isomorphic. Justify your answer by giving an isomorphism if the graphs are isomorphic or stating an invariant property otherwise.



10 Points.

- (b) i. State Euler's Formula for a connected planar graph.

10 Points.

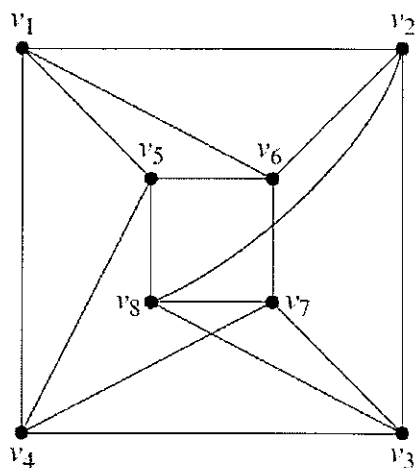
- ii. Suppose that the degree sequence of a connected planar graph is given by 5, 5, 4, 3, 3, 3, 3. Into how many regions is the plane divided by a planar representation of this graph?

15 Points.

- (c) i. State Kuratowski's theorem.

10 Points.

- ii. Use Kuratowski's theorem to show that the following graph is non-planar.



25 Points.

Total: 100 Points.

END