Supervisor's Signature:............................

Date:..............................

University of Kelaniya - Sri Lanka

Centre for Distance & Continuing Education

Bachelor of Science (General) External

First year semester Examination-2024 (September 2025)

(New syllabus)

Faculty of Science

Index Number:..

Course Code:..

Course Title:..

Date:..

QUESTION	MARKS	
	First Examiner	Second Examiner
1		
2		
3		
4		
5		
6		
TOTAL		

First Examiner's Marks, %	
Second Examiner's Marks, %	
Total (200)	
Average	

PURE MATHEMATICS

PMAT 16513 – Discrete Mathematics I

No. of Questions: Six (06) **No. of Pages: Thirteen (13)** **Time Allowed: Two and Half ($2\frac{1}{2}$) hrs**

Instructions to candidates

Programmable Calculators Are Not Allowed

Answer only **Five (05)** Questions. **Question 01** is **compulsory** and **FOUR** other questions should be attempted from the rest of five questions.
All questions are to be **answered in the boxes** provided within this booklet.
You are **not allowed** to remove any page from this booklet.

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1. Answer **ALL** the following multiple-choice questions by writing the letter corresponding to the correct answer, in the table provided below.
Note: Only one letter can be written in each box for each question.
[10 marks for each correct answer – Total for Question 1 is 100 marks]

Question No.	Answer
(i)	
(ii)	
(iii)	
(iv)	
(v)	

Question No.	Answer
(vi)	
(vii)	
(viii)	
(ix)	
(x)	

(i) The propositions p, q, r and s are defined as follows:
p: they play cards, q: they quarrel, r: they run, s: they sleep

What is the correct statement for the following expression?
$$(p \wedge \sim r) \leftrightarrow \sim (q \vee s)$$

A. They play cards and don't run if and only if they neither quarrel nor sleep.
B. They play cards and don't run if and only if they do not quarrel or do not sleep.
C. They play cards and don't run if and only if they do not quarrel and sleep.
D. None of these.

(ii) Which of the following propositions is a tautology?

A. $(p \vee q) \rightarrow p$ **B.** $p \vee (q \rightarrow p)$ **C.** $p \vee (p \rightarrow q)$ **D.** $p \rightarrow (q \rightarrow p)$

(iii) The proposition $p \wedge (\sim p \vee q)$ is

A. a tautology **B.** a contradiction **C.** logically equivalent to $p \wedge q$ **D.** none of these

Continued…

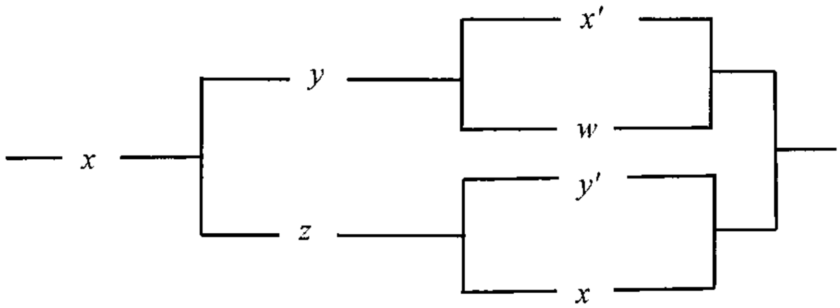
(iv) Let $P(S)$ denote the power set of a set S. Which of the following is always true?

A. $P(S) \cap P(P(S)) = \{\phi\}$ **B.** $P(P(S)) = P(S)$ **C.** $P(S) \cap S = P(S)$ **D.** $S \notin P(S)$

(v) Let A and B be non-empty sets of a universal set U and A^c and B^c denote the complements of the sets A and B. The set $(A \cup B) \setminus (A \cap B)$ is equal to

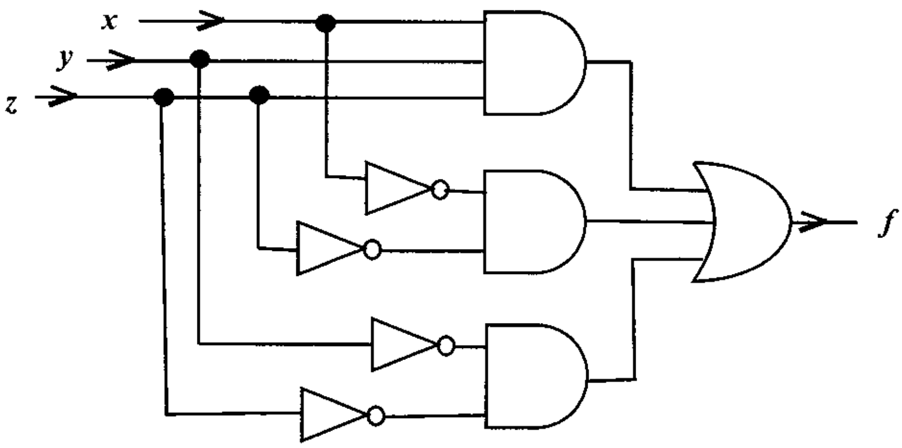
A. $(A \setminus B) \cap (B \setminus A)$ **B.** $(A \setminus B) \cup (B \setminus A)$ **C.** $A \cup B$ **D.** $A^c \cup B^c$

(vi) The switching function, *f*, for the following network is given by:



A. $f(x, y, z, w) = x * [(y + z) * (x' + w + y' + x)]$
B. $f(x, y, z, w) = x + [y + (x' * w) * z + (y' * x)]$
C. $f(x, y, z, w) = x + \{y + (x' * w) * [z + (y' * x)]\}$
D. $f(x, y, z, w) = x * [y * (x' + w) + z * (y' + x)]$

(vii) The Boolean expression, *f*, corresponding to the following combinatorial circuit in terms of inputs *x*, *y* and *z* is given by



A. $(x \vee y \vee z) \wedge (\overline{x} \vee \overline{z}) \wedge (\overline{y} \vee \overline{z})$ **B.** $(x \wedge y \wedge z) \vee (\overline{x} \wedge \overline{z}) \vee (\overline{y} \wedge \overline{z})$
C. $(x \wedge y \wedge z) \vee \overline{(x \wedge z)} \vee \overline{(y \wedge z)}$ **D.** $(x \wedge y \wedge z) \vee \overline{(x \wedge z) \wedge (y \wedge z)}$

(viii) If $A = \{1, 2, 3, 11, 22, 33, 44, \ldots, 99\}$ and $B = \{2, 3, 7, 9\}$, then the number of elements in $(A \times B) \cap (B \times A)$ is

A. 4 **B.** 36 **C.** 40 **D.** 44

Continued...

(ix) Let R_1 and R_2 be two equivalence relations on a set. Consider the following assertions:

(a) $R_1 \cup R_2$ is an equivalence relation

(b) $R_1 \cap R_2$ is an equivalence relation

Then,

A. both assertions are true. **B.** assertion (a) is true, but assertion (b) is not true.
C. both assertions are false. **D.** assertion (b) is true, but assertion (a) is not true.

(x) Which of the following is false?

A. Two mappings f and g are equal if and only if f and g have the same domain A and $f(x) = g(x), \forall x \in A$.
B. The inverse of a one-one function is one-one and onto.
C. The mapping $f: X \longrightarrow X : f(x) = {}^1/_x, \forall x \in X$, where $X = \{x \in \mathbb{R}, x \neq 0\}$, is one-one but not onto.
D. Let $f: S \longrightarrow T,\ g: T \longrightarrow U$ be functions and S, T & U, non-empty sets. If each f and g is one-one, then so is $g \circ f$.

2. **(i)** Let p, q and r be propositions. Then, using the truth table, show that $[p \Rightarrow (q \wedge r)]$ and $[(p \Rightarrow q) \wedge (p \Rightarrow r)]$ are logically equivalent.

[20 marks]

(ii) Stating all the steps clearly, determine the truth value of each of the following statements, where x and y are all integers.

(a) $\exists x\ \forall y: y = x^2 + 2x + 1$, **[10 marks]**

Continued...

(b) $\forall x \, \exists y : \, y^2 - x < 2022,$ **[10 marks]**

(c) $\exists y \, \exists x: 18x + 30y = 185.$ **[15 marks]**

(iii) When you visit the embassy to get visa, a visa officer 1 at the counter Mr. Hans answers as "if you are eligible for student visa, then your age must be below 18. If you are not below 18, then you do not qualify for a scholarship."
Further, a visa officer 2 at the same counter Mr. Erick answers as "if you qualify for a scholarship, then you are eligible for student visa."

(a) Write down the answers given by Hans and Erick above in symbolic notation taking propositions p, q and r as 'you are eligible for student visa', your age is below 18' and 'you qualify for a scholarship' respectively. **[15 marks]**

(b) Use a truth table to find the validity of the answer given by Erick based on the answer given by Hans.

[30 marks]

Continued...

3. **(i)** By using proof by contraposition method, show that if $3n + 2$ is odd then n is odd. **[20 marks]**

(ii) By using proof by contradiction method, show that if $x, y \in \mathbb{Z}$ then $x^2 - 4y \neq 2$. **[25 marks]**

Continued...

(iii) Let a_n be a sequence defined by $a_1 = 1, a_2 = 4, a_3 = 9$ and $a_n = a_{n-1} - a_{n-2} + a_{n-3} + 2(2n - 3), \forall n \geq 4$. Prove by using the principle of strong induction that $a_n = n^2$ is true for all positive integers n. **[30 marks]**

(iv) Show that if x is a real number such that $\frac{x^2-1}{x+2} > 0$, then either $x > 1$ or $-2 < x < -1$. **[25 marks]**

Continued...

4. **(i) (a)** By clearly stating the laws on operations on sets, prove that $(A \cup B)\backslash(A \cap B) = (A\backslash B) \cup (B\backslash A)$; where A and B are non-empty sets. **[25 marks]**

(b) For any non-empty sets A and B, prove that $(A\backslash B) \cup (B\backslash A) = (A \cup B)\backslash(A \cap B)$. **[25 marks]**

Continued...

(ii) Let R be the relation such that $R = \{(x, y): 4 \mid (x + 3y), x, y \in \mathbb{Z}\}$ defined on $\mathbb{Z}$.

(a) Show that R is an equivalence relation on $\mathbb{Z}$. **[35 marks]**

(b) Determine the distinct equivalence classes of R . **[15 marks]**

5. **(i)** Consider the mapping $f: X \longrightarrow Y$, where X and Y are non-empty sets.
Write down all the conditions that should satisfy by f for the existence of the inverse of f.
[15 marks]

Continued...

(ii) Show that the following function is one-one and onto. Also find a formula that defines the inverse function.

$f: \mathbb{R}_0 \longrightarrow \mathbb{R}_0 : f(x) = \frac{1}{x}, \forall\, x \in \mathbb{R}_0$, where $\mathbb{R}_0 = \mathbb{R}\backslash\{0\}$. **[35 marks]**

(iii) Let $f: X \longrightarrow Y$ and $g: Y \longrightarrow X$. If $g \circ f$ is an identity function on X and $f \circ g$ is an identity function on Y, then, show that

(a) f is one-one **[15 marks]**

Continued...

(b) f is onto **[20 marks]**

(c) $g = f^{-1}$. **[15 marks]**

6. (i) Let $X = \{1, 2, 3\}$, $A = \{1, 2\}$, and $A' = \{3\}$. Show that the set $S = \{X, A, A', \emptyset\}$ along with operations $\cup$ and $\cap$ forms a Boolean algebra. **[35 marks]**

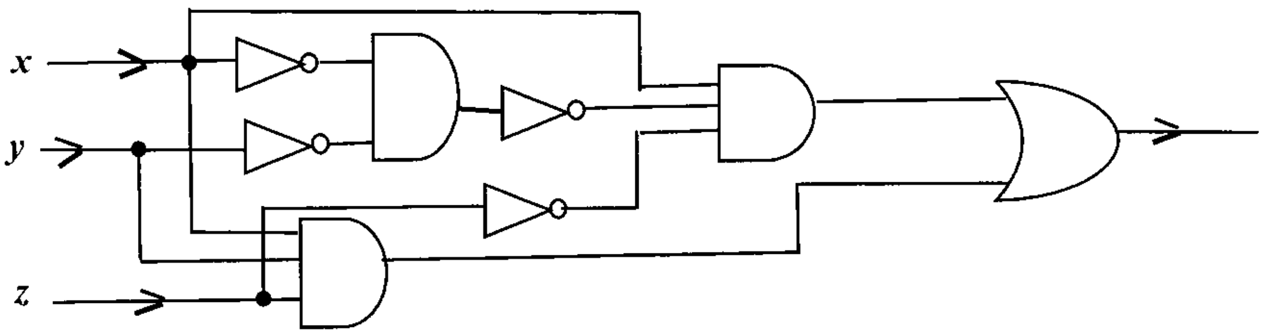
Continued...

(ii) Let x, y, z be elements of a Boolean algebra. Prove that if $x + y = x + z$ and $x' + y = x' + z$, then $y = z$.

[20marks]

(iii) (a) Find the output of the following combinatorial circuit in terms of inputs x, y and z.

[25 marks]



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(b) Draw a combinatorial circuit by simplifying the Boolean expression obtained in part (a).
[20 marks]

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