University of Kelaniya- Sri Lanka
Faculty of Science
Centre for Distance & Continuing Education
Bachelor of Science (General) Degree
First Examination-External
September 2025

PURE MATHEMATICS | **PMAT 16522 - Matrix Algebra**

No. of Questions: Five (05) **No. of Pages:** Four (04) **Time:** Two (02) hours

Answer only FOUR (04) questions.

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1. (a) Let $A = \begin{bmatrix} -4 & 0 \\ 1 & -5 \\ -3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ -2 & 1 \\ 4 & 4 \end{bmatrix}$. Solve the matrix equation $2A - 5B = 3X$ for matrix X.

[20 marks]

(b) Use the inverse matrices $A^{-1} = \begin{bmatrix} 2 & 5 \\ -7 & 6 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} 7 & -3 \\ 2 & 0 \end{bmatrix}$ to find the following:

i. $(AB)^{-1}$

[10 marks]

ii. $(A^T)^{-1}$

[10 marks]

iii. $(2A)^{-1}$

[10 marks]

(c) Let A be a square matrix.

i. Prove that $A^2 = A$ if and only if $(A^T)^2 = A^T$.

[20 marks]

ii. Prove that if $A^2 = A$, then either $A = I$ or A is singular.

[15 marks]

iii. Determine conditions on a and b such that $A^2 = A$, where

$$A = \begin{bmatrix} 1 & 0 \\ a & b \end{bmatrix}.$$

[15 marks]

Continued.

2. (a) Determine the polynomial function. $P(x) = a_3x^3 + a_2x^2 + a_1x + a_0$ whose graph passes through given points, $(-1, 3), (0, 0), (1, 1)$ and $(4, 58)$.

[40 marks]

(b) Find value(s) of k such the following system of linear equations has no solution.

$$\begin{aligned} x + ky &= 1 \\ kx + y &= 0. \end{aligned}$$

[20 marks]

(c) Find values of a, b and c such the following system of linear equations,

$$\begin{aligned} ax + by + 2z &= c \\ 4x - 2y + 5z &= 0 \\ x + y &= 0, \end{aligned}$$

i. has a unique solution, and

[30 marks]

ii. have infinitely many solutions.

[10 marks]

3. (a) Consider the following matrix A,

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 4 \end{bmatrix}$$

(i) Find **elementary matrices** E_1, E_2 and E_3 such that $E_3E_2E_1A = I$, where I is the identity matrix of order 3.

[30 marks]

(ii) Using part (i) above, factorize A into a product of elementary matrices.

[10 marks]

(iii) Find A^{-1} using the elementary matrices E_1, E_2 and E_3.

[10 marks]

(b) Consider the following linear system of equations;

$$\begin{aligned} x + 3y - 2z &= -1 \\ 2x + 5y + z &= 2 \\ 2x + 6y - 4z &= -2 \end{aligned}$$

Continued.

i. Write the linear system in the form $Ax = b$.

[05 marks]

ii. Compute $\det(A)$.

[10 marks]

iii. Does the system have a unique solution? Explain.

[10 marks]

iv. Find all solutions to the system.

[25 marks]

4. (a) Use the properties of determinants to show that

$$\begin{vmatrix} a & 1 & 1 & 1 \\ 1 & a & 1 & 1 \\ 1 & 1 & a & 1 \\ 1 & 1 & 1 & a \end{vmatrix} = (a+3)(a-1)^3$$

[40 Marks]

(b) Find the area of the triangle having the vertices at $(1,1), (-1,1)$ and $(0,-2)$.

[10 Marks]

(c) Let $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ and $\det(A) = -8$. Find the values of the following:

i. $\det(2A^{-1})$

[10 marks]

ii. $\begin{vmatrix} a & g & d \\ b & h & e \\ c & i & f \end{vmatrix}$

[15 marks]

(d) Let $B = \begin{bmatrix} -4 & -4 & 4 \\ -1 & 0 & 1 \\ -7 & -6 & 7 \end{bmatrix}$. If I is the 3×3 identity matrix, then determine the values of the constant λ, so that $B + \lambda I$ is singular.

[25 Marks]

Continued.

5. (a) Consider the matrix $B = \begin{bmatrix} a & 1 & b \\ c & 7 & 0 \\ 3 & d & 2 \end{bmatrix}$. If $u = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$ and $v = \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}$ are eigenvectors of B with the corresponding eigenvalues λ and μ, then determine the values of a, b, c, d, λ and μ.

[40 Marks]

(b) If C is an $n \times n$ matrix, show that C and C^T have the same eigenvalues.

[20 Marks]

(c) Let $A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$, where a and b are nonzero real numbers.

i. Find the eigenvalues of A.

[10 marks]

ii. Find the corresponding eigenvectors of A.

[20 marks]

iii. Find a nonsingular matrix P and a diagonal matrix D so that $P^{-1}AP = D$.

[10 marks]

*****End of the Paper*****