University of Kelaniya- Sri Lanka
Faculty of Science
Centre for Distance & Continuing Education
Bachelor of Science (General) Degree
First Examination-External
May 2025
Academic Year 2023- Semester II

PURE MATHEMATICS | **PMAT 17532 - Discrete Mathematics II**

No. of Questions: Five (05) **No. of Pages:** Three (03) **Time:** Two (02) hours

Answer only FOUR (04) questions.

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1. (a) i. Prove that if $a \mid b+c$ and $a \mid b$ then $a \mid c$.
   ii. Show that if m and n are both even or m and n are both odd then 4 divides $m^2 - n^2$.

   (b) i. If the greatest common divisor of a and b is 1, find the greatest common divisor of $a+b$ and $a-b$.
   ii. Prove that the greatest common divisor of two integers a and b, not both zero, is the least positive integer d such that $d = ma + nb$, for some integers m and n.
   iii. Find the greatest common divisor d, of 621 and 484, and express d as a linear combination of these two numbers.

   (c) Prove that for given any integer n, there exists n consecutive composite integers.

2. (a) Let m be a positive integer. Define what is meant by saying that a is congruent to b modulo m.

   i. Prove that if $a \equiv b \pmod{m}$ then $a^2 \equiv b^2 \pmod{m}$. What can you say about the statement $a^2 \equiv b^2 \pmod{m}$ imples $a \equiv b \pmod{m}$?
   ii. Prove that $x \equiv 2 \pmod{9}$ if and only if $5x \equiv 10 \pmod{9}$.
   iii. Find the remainder when 2^{20} is divided by 7.

   (b) i. If s is a solution to the linear congruence $ax \equiv b \pmod{m}$ and $s \equiv t \pmod{m}$, then show that t is also a solution to the congruence $ax \equiv b \pmod{m}$.
   ii. Find all incongruent solutions of x satisfying the equation $35x \equiv 10 \pmod{45}$.

3. (a) State the condition on a, b and c such that the equation $ax + by = c$ has integer solutions.

   Find all integer solutions to the following linear Diophantine equations if any

   i. $8x + 12y = 5$
   ii. $18x + 30y = 6$

Continued.

(b) i. Use the **Generalized Pigeonhole Principle** to find the minimum number of students, each of whom comes from one of the 30 cities in Colombo, who must be enrolled in University of Kelaniya to guarantee that there are at least 150 who come from the same city?

ii. How many strings of **eight** uppercase English letters are there that start with **X**, if letters can be repeated?

(c) i. State the **binomial theorem.**

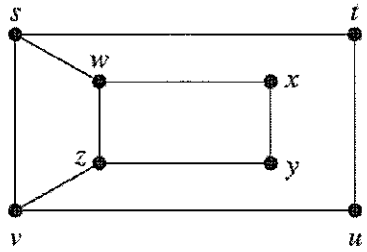
ii. Using the **binomial theorem** find the coefficient of x^6y^9 in the expansion of $(5x^2 + 2y^3)^6$.

4. (a) i. State the **Handshaking theorem.**

ii. Prove that in a simple graph with at least two vertices there must be two vertices that have the same degree.

iii. Suppose that the degree sequence of a simple graph G is given by $4, 3, 3, 2, 2$. How many edges does $\overline{G}$ have?

(b) Consider the following graph.

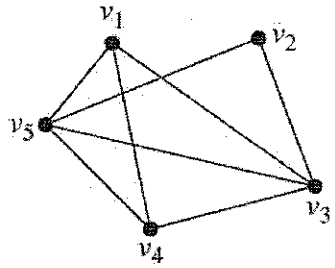


i. Does the graph contain an **Euler cycle**? **Justify your answer.**

ii. Does the graph contain a **Hamilton cycle**? **Justify your answer.**

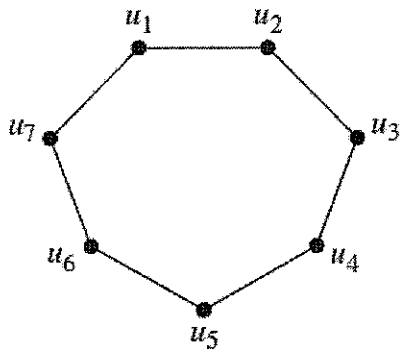
iii. Draw the subgraph **induced** by the vertex set $\{s, w, v, y\}$.

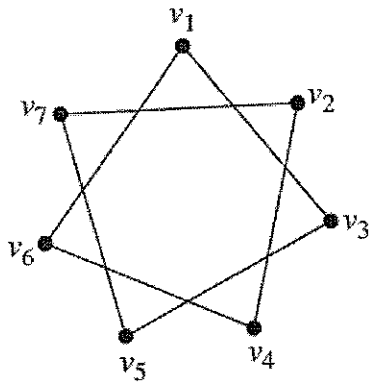
(c) Find the **chromatic number** of the below graph.



Continued.

5. (a) Determine whether the following pair of graphs is isomorphic. Justify your answer by **giving an isomorphism** if the graphs are isomorphic or **stating an invariant property** otherwise.



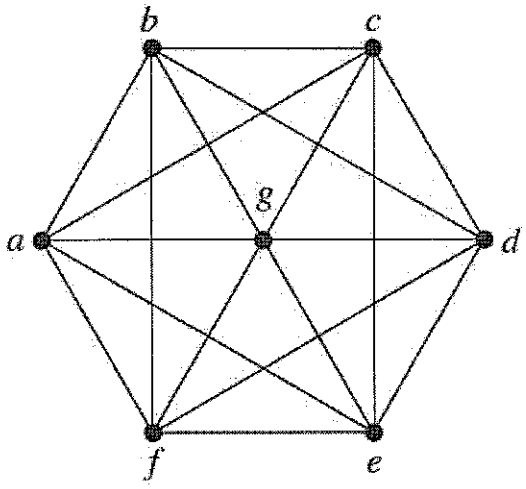


(b) i. State **Euler's Formula** for a connected planar graph.

ii. Suppose that a connected planar graph has **six vertices, each of degree four.** Into how many **regions** is the plane divided by a planar representation of this graph?

(c) i. State **Kuratowski's theorem.**

ii. Use Kuratowski's theorem to show that the following graph is non-planar.



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