University of Kelaniya – Sri Lanka
Centre for Distance & Continuing Education
Bachelor of Science (General) External
First year second semester repeat examination - 2019 (2024 August)
(New Syllabus)
Faculty of Science
Pure Mathematics
PMAT 17543- Theory of Calculus

No. of Questions: Six (06) No. of Pages: Two (02) Time: Two & half (2 1/2) Hours.

Answer **Five (05)** questions only.

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1. (a) Let A be a non-empty bounded subset of $\mathbb{R}$. In the usual notations, prove that
$$\sup(kA) = \begin{cases} k \sup A, & k \geq 0 \\ k \inf A, & k < 0 \end{cases}$$

(b) Let S be a nonempty bounded subset of $\mathbb{R}$ and let $-S = \{-s : s \in S\}$. Prove that $\inf S = -\sup(-s)$.

(c) Find the supremum and infimum of the set $A = \left\{n + \frac{(-1)^n}{n} \middle| n \in \mathbb{N}\right\}$, if they exist.

(d) Use the rational zero test to find the rational zeros of $3x^3 - 5x^2 + 4x + 2$.

2. (a) Use $\varepsilon - \delta$ definition to show that $\lim_{x \to 1} x^2 - 4x + 5 = 1$.

(b) Find the value of a so that $\lim_{x \to 1} f(x)$ exist, where $f(x) = \begin{cases} 3x + 5, & x \leq 1 \\ 2x + a, & x > 1 \end{cases}$

(c) Use the L'Hospital's rule to evaluate the following limits:

(i) $\lim_{x \to \infty} x^{1/x}$ (ii) $\lim_{x \to -\infty} xe^x$ (iii) $\lim_{x \to \infty} \frac{6 - 6\cos x - 3\sin^2 x}{x^3}$

3. (a) Show that the following function f is continuous everywhere on $\mathbb{R}$.
$$f(x) = \begin{cases} \frac{1 - \ln(1 - 2x)}{x}, & x < 0 \\ 2, & x = 0 \\ \frac{\sin 2x}{x}, & x > 0 \end{cases}$$

(b) Find the value of k such that the function $f(x) = \begin{cases} \frac{x^2 - 16}{x - 4}, & x \neq 4 \\ k, & x = 4 \end{cases}$ is continuous at $x = 4$.

(c) State the Intermediate Value Theorem (IVT).
Using IVT, show that $25 - 8x^2 - x^3 = 0$ has at least one root in the interval [-2,4].

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4. (a) Define the differentiability of a function $f(x)$ at $x = a$.

(b) Let f be a function defined as follows:

$f(x) = \begin{cases} x^2 + (2c+1)x - p, & x \geq -1 \\ e^{2x+2} + cx + 3q, & x < -1 \end{cases}$, where p and q are constants. If f is differentiable at $x = -1$, then what is the value of $p - q$?

(c) State the Rolle's Theorem and the Mean Value Theorem.
Using the Mean Value Theorem, show that $e^x \geq 1 + x$ for all $x \in \mathbb{R}$.

(d) Find the slope of the tangent line to the curve described by
$2(x^2 + y^2)^2 = 25(x^2 - y^2)$ at the point $(3,1)$.

5. Let $y = f(x) = \frac{2x^2}{x^2-1}$.

(a) Find x and y intercepts of f. Also find the vertical and horizontal asymptotes of f if there are any.
(b) Find the critical points of f and use the second derivative test to classify them.
(c) Determine the concavity of the graph of f and hence find the points of inflection if there are any.
(d) Sketch the graph of f.

6. (a) Find the arc length of the curve $y = \frac{2x^{2/3}}{3}$ for $1 \leq x \leq 2/3$.

(b) Use the cylindrical shell method to find the volume of the solid obtained by rotating the region bounded by $x = y^2 - 4$ and $x = 6 - 3y$ about the line $y = -8$.

(c) Let $f(x) = 2\sqrt{x}$ over the interval $[1,2]$. Find the surface area of the surface generated by revolving the graph of $f(x)$ around x-axis.

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