University of Kelaniya- Sri Lanka
Faculty of Science
Centre for Distance & Continuing Education
Bachelor of Science (General) Degree
First Examination-External
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PURE MATHEMATICS | **PMAT 16522- Matrix Algebra**

No. of Questions: Five (05) **No. of Pages:** Two (02) **Time:** Two (02) hours

Answer only FOUR (04) questions.

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1. (a) Let $A = \begin{bmatrix} -2 & -1 \\ 1 & 0 \\ 3 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 3 \\ 2 & 0 \\ -4 & -1 \end{bmatrix}$. Solve the matrix equation $2A+4B = -2X$ for matrix X.

   (b) Prove that if A and B are $n \times n$ **skew-symmetric matrices**, then $A+B$ is skew-symmetric.

   (c) i. Find A provided that $(2A)^{-1} = \begin{bmatrix} 2 & 4 \\ -3 & 2 \end{bmatrix}$.

   ii. Find the inverse of the matrix A where $A = \begin{bmatrix} 2 & -17 & 11 \\ -1 & 11 & -7 \\ 0 & 3 & -2 \end{bmatrix}$.

2. (a) Factor the following matrix into a product of **elementary matrices.**

$$A = \begin{bmatrix} 1 & -2 & 0 \\ -1 & 3 & 0 \\ 0 & 2 & 1 \end{bmatrix}.$$

   (b) Find value(s) of k such the following system of linear equations,

$$\begin{aligned} kx + 2ky + 3kz &= 4k \\ x + y + z &= 0 \\ 2x - y + z &= 1, \end{aligned}$$

   i. has a unique solution, and

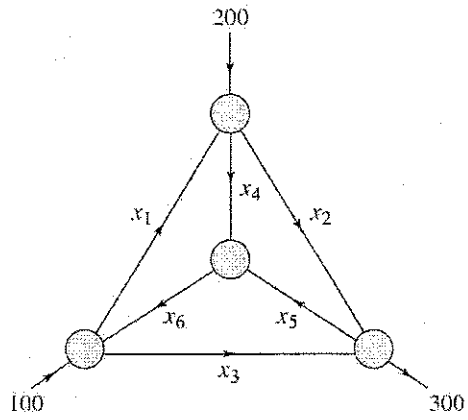
   ii. have infinitely many solutions.

   (c) Find value(s) of k such the following system of linear equations has no solution,

$$\begin{aligned} x + ky &= 2 \\ kx + y &= 4. \end{aligned}$$

Continued.

3. (a) The flow of traffic (in vehicles per hour) through a network of streets is shown in the following figure.



i. Solve the system for the traffic flow represented by $x_i,\ i = 1, 2, \cdots, 6$.
ii. Find the traffic flow when $x_3 = 100, x_5 = 50$, and $x_6 = 50$.

(b) Solve the following system of linear equations using Cramer's Rule:

$$\begin{aligned} 2x + y - z &= 3 \\ x + z &= 2 \\ x + y &= 1 \end{aligned}$$

4. (a) Using the properties of determinants, show that

$$\begin{vmatrix} a & b & c \\ b+c & c+a & a+b \\ a^2 & b^2 & c^2 \end{vmatrix} = -(a-b)(b-c)(c-a)(a+b+c).$$

(b) Let $A = (1, 2)$ and $B = (4, 6)$ be two points. Using a determinant, find the equation of the line passing through the points A and B.

(c) Determine whether the vectors $v_1 = (1, -2, 0), v_2 = (-1, 3, 1), v_3 = (0, -1, 2)$ are linearly dependent or not.

(d) Let $A = \begin{bmatrix} 1 & -3 \\ 2 & 4 \end{bmatrix}$. Show that for all $k \in \mathbb{R}$, the matrix $A + kI$ is non singular.

5. (a) Let A be a square matrix of size n with the property $A^2 = 3A$. Show that the only possible eigenvalues of A are 0 and 3.

(b) Let $A = \begin{bmatrix} 1 & -2 & 2 \\ -2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$.

i. Find the characteristic equation of A.
ii. Show that $\lambda = 5$ is an eigenvalue of A.
iii. Find the other eigenvalue(s) of A and all corresponding eigenvectors of A.
iv. Is A diagonalizable? If so, find matrices P and D with $P^{-1}AP = D$.

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