University of Kelaniya – Sri Lanka
Centre for Distance and Continuing Education

Bachelor of Science (External) First Year First Semester - 2023
2024 – November
Faculty of Science

Pure Mathematics
PMAT 16513 – Discrete Mathematics I

No. of Questions: Six (06)   No. of Pages : Three (03) Time Allowed : Two & half (2 ½) hrs.

Answer Five (05) Questions Only

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1. (i) By clearly stating the Laws of Sets you use, prove that $(A \cap B)\backslash(A \cap C) = A \cap (B\backslash C)$, where A, B and C are non-empty sets. **[25 marks]**

(ii) For any non-empty sets A and B, prove that $(A\backslash B) \cap B = \phi$. **[25 marks]**

(iii) Let R be the relation such that $R = \{(a,b): a \equiv b(mod\ 3)\}$ on the set of integers, $\mathbb{Z}$.

(a) Show that R is an equivalence relation on $\mathbb{Z}$. **[35 marks]**

(b) Determine the equivalence classes of R . **[15 marks]**

2. (i) Define the followings for the rule $f: X \longrightarrow Y$, where X and Y are non-empty sets.

(a) f is a function from X to Y. **[05 marks]**

(b) f is a one-one function. **[05 marks]**

(c) f is an onto function **[05 marks]**

Continued…

(ii) Show that the following function is one-one and onto. Find a formula that defines the inverse function.

$f: \mathbb{Q} \longrightarrow \mathbb{Q} : f(x) = 3x + 5, \forall\, x \in \mathbb{Q}$, where $\mathbb{Q}$ is the set of all rational numbers.

[35 marks]

(iii) Let $f: X \longrightarrow Y$ be a function and I_X and I_Y be the identity functions on X and Y respectively. Then, show that $I_Y \circ f = f = f \circ I_X$. **[30 marks]**

(iv) Given $g = \{(1, b), (2, c), (3, a)\}$ a function from $X = \{1, 2, 3\}$ to $Y = \{a, b, c, d\}$, and $f = \{(a, x), (b, x), (c, z), (d, w)\}$, a function from Y to $Z = \{w, x, y, z\}$, write $f \circ g$ as a set of ordered pairs and draw the arrow diagram of $f \circ g$. **[20 marks]**

3. (i) Let p, q and r be propositions. Using the truth table, show that $[p \Rightarrow (q \wedge r)]$ and $[(p \Rightarrow q) \wedge (p \Rightarrow r)]$ are logically equivalent. **[20 marks]**

(ii) Stating all the steps clearly, determine the truth value of each of the following statements, where x and y are real numbers.

(a) $\forall x\, \exists y: y = -2x^2 + x + 5,$ **[10 marks]**

(b) $\exists y\, \forall x: y = -2x^2 + x + 5,$ **[10 marks]**

(c) $\exists y\, \forall x: (3x - y)^2 = 9x^2 + y^2$ **[15 marks]**

(iii) There are two shopping malls next to each other, one with sign board as 'Good items are not cheap' and second with sign board as 'Cheap items are not good'.

(a) Write down the above two statements in sign boards using symbolic notation taking propositions p and q as 'Items are good' and 'Items are cheap' respectively. **[20 marks]**

(b) Use a truth table to examine whether the sign boards express the same meaning. **[25 marks]**

4. (i) Prove that $n^2 + n$ is even for any integer n using the 'proof by cases'. **[25 marks]**

(ii) Let $P(n)$ be the statement that $1 + \frac{1}{4} + \frac{1}{9} + \ldots + \frac{1}{n^2} < 2 - \frac{1}{n}$, where n is an integer greater than 1.

(a) What is the statement $P(2)$? **[05 marks]**

(b) Show that $P(2)$ is true, completing the basic step of the proof in mathematical induction.

[05 marks]

(c) What is the inductive hypothesis? **[05 marks]**

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(d) What do you need to prove in the inductive step? **[10 marks]**

(e) Complete the inductive step. **[20 marks]**

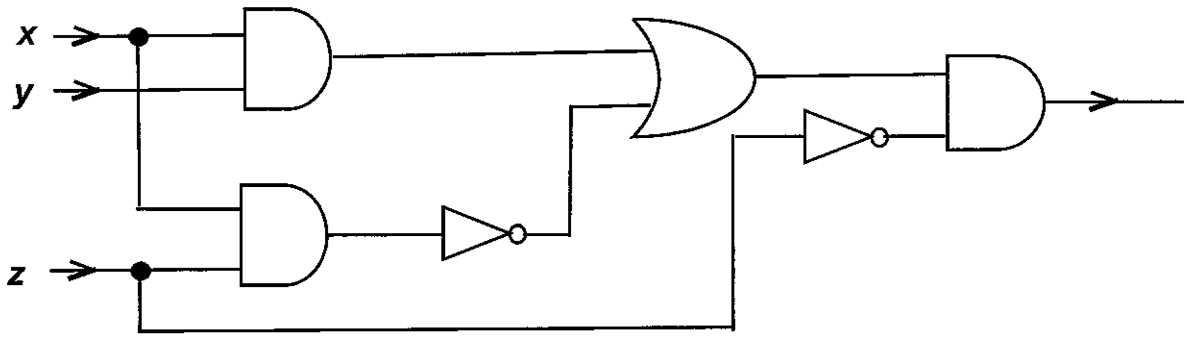
(f) Explain why these steps show that this inequality is true whenever n is an integer greater than 1. **[05 marks]**

(iii) Show that if n is an integer and $n^3 + 5$ is odd, then n is even, using a proof by contraposition (indirect proof). **[25 marks]**

5. (i) Build the combinatorial circuit corresponding to the Boolean expression $(x \wedge \overline{y} \wedge z) \vee (x \wedge y \wedge \overline{z}) \vee (x \wedge \overline{y} \wedge \overline{z})$ when given inputs x, y and z. **[35 marks]**

(ii) Find the output of the following combinatorial circuit in terms of inputs x, y and z.

[30 marks]



(iii) Find the sum-of-products expansion for the Boolean function $F(x, y, z) = (x + y)\bar{z}$. **[35 marks]**

6. (i) Let f be a function from the set of real numbers to the set of nonnegative real numbers with $f(x) = |x|$.

Is f invertible, and if it is, what is its inverse? If f is not invertible, then explain the reason. Also, explain if there is any step that can be taken so that f to be invertible. **[25 marks]**

(ii) Let X, Y and Z be any non-empty sets and let f and g be one-one and onto functions from X to Y and from Y to Z respectively. Then, show that

(a) $g \circ f$ is one-one, **[25 marks]**

(b) $g \circ f$ is onto, **[25 marks]**

(c) $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$. **[25 marks]**

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