University of Kelaniya – Sri Lanka

Centre for Distance & Continuing Education

Bachelor of Science (General) External

First Year Second Semester Examination – 2019

Faculty of Science

Pure Mathematics

PMAT 17532 – Discrete Mathematics II

No. of Questions: Five (05) No. of Pages : **Three** (03) Time : Two(02) hours

Answer **Four (04)** Questions only

Q1) (a) (i) Show that $\frac{n(n+1)(2n+1)}{6}$ is an integer for any $n \in \mathbb{N}$.

(ii) Suppose that $a, b, c, m, n \in \mathbb{Z}$ and $c|a$ and $c|b$. Prove that $c|ma + nb$.

(b) Prove that the greatest common divisor of two nonzero integers a and b is the least positive integer d such that $d = ma + nb$, for some integers m and n.

Find the greatest common divisor d of 360 and 210 and express d as a linear combination of these two integers.

(c) Show that there are infinitely many primes.

Q2) Let m be a positive integer. Define what is meant by saying that a is congruent to b modulo m.

(a) (i) Let a, b, c, d and m be integers such that $m > 0$. Show that if $a \equiv b(mod\ m)$ and $c \equiv d\ (mod\ m)$ then $ac \equiv bd(mod\ m)$.

(ii) If s is a solution to the linear congruence $ax \equiv b(mod\ m)$ and $s \equiv t\ (mod\ m)$, then show that t is also a solution to the congruence $ax \equiv b(mod\ m)$.

(iii) Find all solutions in integers x to $20x \equiv 15\ (mod\ 65)$.

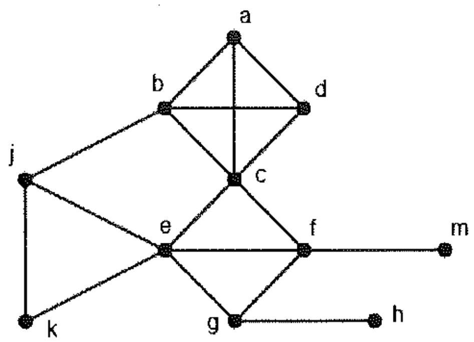
(b) (i) Find the remainder when 5^{123} is divided by 7.

(ii) Show that $3^{1000} + 3$ is divisible by 28.

Q3) (a) State the condition that the equation $ax + by = c$ has integer solutions.

Find all integer solutions of the linear Diophantine equation $60x + 33y = 9$.
Find all positive solutions (if any) to the above equation.

(b) Using Well-Powell algorithm, determine the number of colors you need to color the following graph:



What is the Chromatic number? Explain your answer.

Q4) (a) State and prove the Handshaking lemma for any given graph G.

For each of the following cases, draw a simple graph having the given properties or explain why no such graph exists.

(i) Six vertices with degrees 3,3,4,4,5,5.

(ii) Six vertices with degrees 1,2,2,3,4,5.

(b) Define what is meant by saying a graph $G = (V, E)$ is bipartite.

Prove that the size of a bipartite graph of order n is at most $\frac{n^2}{4}$.

(c) Define an Euler cycle and a Hamiltonian cycle of a graph.

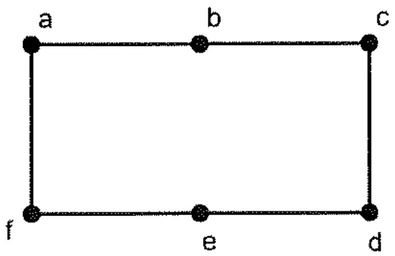
What values of n for which the complete bipartite graph $K_{m,n}$ has

(i) an Euler cycle.

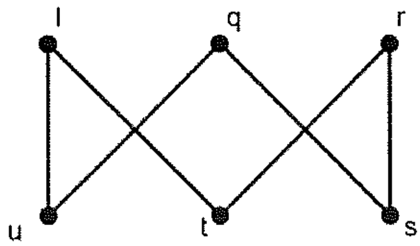
(ii) a Hamiltonian cycle.

Q5) (a) Define what is meant by saying that two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ are isomorphic. Determine whether each of the following pairs of graphs is isomorphic. If any pair is isomorphic, then write down an isomorphism and if they are not isomorphic then state an invariant property one graph possesses but the other does not.

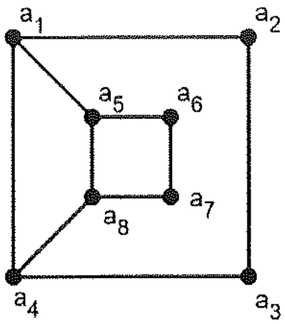
(i) G_1:



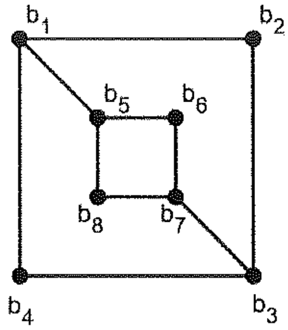
G_2:



(ii) H_1:



H_2:

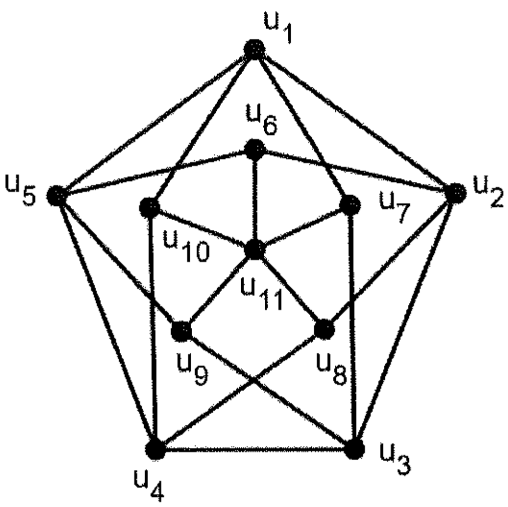


(b) State the Euler's formula for a connected planar graph with v vertices, e edges and f faces.

Draw $K_{2,3}$ as a planar graph and verify the Euler's formula.

(c) State the Kuratowski's theorem.

Determine whether the following graph is planar or not.



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