University of Kelaniya – Sri Lanka
Centre for Distance & Continuing Education
Bachelor of Science (General) External
First year First semester examination - 2019
(New Syllabus)
2022 August
Faculty of Science

PURE MATHEMATICS
PMAT 16522 –Matrix Algebra

No. of Questions: Five (05) **No. of Pages:** Three (03) **Time:** Two (02) hours
Answer **Four (04)** Questions Only.

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01. (a) Let $A = \begin{pmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -2 & -1 \\ -1 & 5 & 6 \\ 5 & -4 & 5 \end{pmatrix}$. Use elementary row operations to find A^{-1} and B^{-1} if they exist.

(b) Let $A = \begin{pmatrix} 2 & 3 \\ 3 & 2 \end{pmatrix}, B = \begin{pmatrix} x & y \\ y & x \end{pmatrix}$ and $C = \begin{pmatrix} a & b \\ b & a \end{pmatrix}$. If $AB = CA$, find x and y in terms of a and b.

(c) Let A and B be two square matrices of same order.

(i) In the usual notations, show that
$$(AB)^{-1} = B^{-1}A^{-1}$$

(ii) If B is symmetric show that
$$(A^T - B)^T + C(B^{-1}C)^{-1} = A$$

Continued…

02. Consider the system of linear equations

$$\begin{aligned} x - y - z &= 3 \\ 3x + 2y + \lambda z &= 8 \\ 2x - 3y - z &= \mu \end{aligned}$$

(a) For what values of λ and μ, the system has

(i) no solutions,

(ii) unique solutions,

and

(iii) infinitely many solutions?

(b) Find the solutions of the linear system for

(i) $\lambda = 0$ and $\mu = 3$,

(ii) $\lambda = -8$ and $\mu = \frac{31}{5}$

03. Nimal invested Rs 50,000 in three bank saving accounts A, B and C. After one year, he received a total of Rs 3,500 in simple interest from the three investments. The saving accounts A, B and C paid interest rates 6%, 7% and 8% annually, respectively. There was Rs 6,000 more invested in the account A than in the accounts B and C.

Let

x = The amount of money invested in the saving account A.
y = The amount of money invested in the saving account B.
z = The amount of money invested in the saving account C.

(i) Setup a system of linear equations to describe Nimal's portfolio.

(ii) Write the system of equation in part (i) in matrix form.

(iii) Use Gauss-Jordan elimination method to find the amount of money Nimal invested in each account.

04. (a) Use elementary row operations and the properties of determinants to show that

(i) $$\begin{vmatrix} a+x^3 & b+x^2 & c+x \\ a+y^3 & b+y^2 & c+y \\ a & b & c \end{vmatrix} = xy(y-x)(a+b(x+y)+cxy).$$

(ii) $$\begin{vmatrix} (b+c)^2 & b^2 & c^2 \\ a^2 & (c+a)^2 & c^2 \\ a^2 & b^2 & (a+b)^2 \end{vmatrix} = abc(a+b+c)^2.$$

Continued...

(b) Consider the following system of linear equations:

$$\begin{aligned} 5x - y + 4z &= -2 \\ 2x + 2y - 3z &= -1 \\ -3x - y + kz &= 5 \end{aligned}$$

(i) Write the above system in matrix form $A\underline{x} = \underline{b}$.

(ii) Evaluate $\det(A)$.

(iii) Hence, find the value of k which the system does not have a unique solution.

(iv) If $k = 4$, use Cramer's rule to find the solution of the linear system.

05. (a) (i) If A is an invertible matrix, prove that A and A^{-1} have the same eigenvectors and find the relationship corresponding to the eigenvalues.

(ii) Prove that an eigenvalue of a square matrix is zero if and only if the matrix is noninvertible.

(b) Let $A = \begin{pmatrix} 3 & 3 & 1 \\ 3 & 1 & 3 \\ 1 & 3 & 3 \end{pmatrix}$.

(i) Show that the characteristic equations of A is given by $(\lambda - 7)(\lambda - 2)(\lambda + 2) = 0$

(ii) Find the eigenvectors of A corresponding to each eigenvalues.

(iii) Find a nonsingular matrix P and a diagonal matrix D such that $P^{-1}AP = D$.

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