UNIVERSITY OF KELANIYA - SRI LANKA

FACULTY OF SCIENCE

Bachelor of Science (External) Degree Examination, August 2022

Academic Year 2019/2020 - Semester I

PURE MATHEMATICS

PMAT 16513 – Discrete Mathematics I

No. of Questions: Six (06) No. of Pages: Two (02) Time Allowed: Two & half (2 ½) hrs

Answer Five **(05)** Questions Only

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1. (i) Let p, q, and r be three simple propositions.
(a) Construct a truth table for the compound proposition $(\sim p \leftrightarrow \sim q) \rightarrow (q \leftrightarrow r)$
(b) Show that $\sim p \rightarrow (q \rightarrow r)$ and $q \rightarrow (p \vee r)$ are logically equivalent.

[40 marks]

(ii) Determine the truth value of each of the following statements, if the domain consists of all real numbers.
(a) $\exists x(x^3 = -1)$ (b) $\exists x \forall y(y = x^2 + 2x + 1)$ (c) $\exists x \exists y(x^2 + y^2 = 2xy)$ **[30 marks]**

(iii) Formalize the following and by writing truth tables for premises and conclusions, determine whether the following argument is valid.
" If Jane can sing or Davide can play, then I will buy the compact disc. Jane cannot sing. Therefore, I will buy the compact disc." You may assume that p is "Jane can dance", q is "David can play", and r is "I will buy a compact set".

[30 marks]

2. (i) (a) Let A, B and C be three non-empty sets. By stating the laws on operations on sets, prove that
(α) $(A \setminus B)\setminus C = A\setminus(B \cup C)$.
(β) $(A \cup B)\setminus C = (A\setminus C) \cup (B\setminus C)$. **[25 marks]**

(b) Prove that, in the usual notations, $A^c\setminus B = A^c \cap B^c$ by showing that each set is a subset of the other. **[25 marks]**

(ii) Let A, B and C be three non-empty sets. Prove or disprove that if $A \times B = A \times C$, then $B = C$. **[25 marks]**

(iii) Let A, B and C be three non-empty sets. Prove that $A \times (B \cap C) = (A \times B) \cap (A \times C)$. **[25 marks]**

3. (i) Determine whether the following function is one-one
$f: \left[-\frac{1}{2}, \frac{1}{2}\right] \rightarrow \mathbb{R} : f(x) = \frac{x}{1+x^2}$. **[15 marks]**

(ii) Show that the following function is onto.
$f: \mathbb{R} \rightarrow \mathbb{R} : f(x) = 5x - 2$ for $\forall x \in \mathbb{R}$ **[15 marks]**

(iii) Consider the mapping $f: X \longrightarrow Y$, where X and Y are non-empty sets.
(a) Write down conditions that should satisfy by f for the existence of the inverse of f.

(b) Find the inverse of the function $f(x) = \sqrt[4]{6x-7}$. **[40 marks]**

(iv) The functions f and g are given by

$f(x) = \frac{2x+3}{2x-3}$, $x \in \mathbb{R}, x \neq \frac{3}{2}$, $g(x) = x^2 + 2, x \in \mathbb{R}$

Find

(a) the range of $g(x)$

(b) an expression for $f o\, g(x)$

(c) an expression for $f^{-1}(x)$ **[30 marks]**

4. (i) Let B is Boolean algebra and $a, b \in B$. Then prove the followings:

(a) $a'b' + ab + a'b = a' + b$.

(b) $(a'b' + c)(a + b)(b' + ac)' = a'bc$.

(c) $a = 0$ if and only if $b = a.b' + a'.b$ for all b. **[40 marks]**

(ii) Write down the dual of $(x \cdot (x + y \cdot 0))' = x'$ for all x and y in a Boolean algebra. **[10 marks]**

(iii) A relation R on $\mathbb{Z}$ is defined as $xR\,y$ if and only if $3x - 5y$ is even.

(a) Prove that R is an equivalence relation on $\mathbb{Z}$.

(b) Determine the two equivalence classes of R. **[50 marks]**

5. (i) Define the sequence $a_1, a_2, a_3, \ldots$ by $a_1 = 1$, $a_2 = 2$, and $a_n = 2a_{n-1} - a_{n-2}$ for all $n \geq 3$. By using strong mathematical induction prove that $a_n = n$ for all $n \in \mathbb{N}$. **[30 marks]**

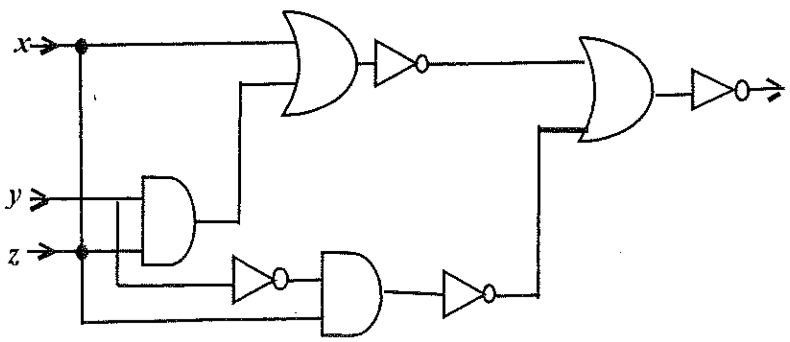
(ii) By using the method of contradiction, prove that for $a, b \in \mathbb{Z}$ and $a \geq 2$, either a not divides b or a not divides $b + 1$. **[25 marks]**

(iii) By using the method of contrapositive, show that for $x, y \in \mathbb{R}$, if $y^3 + yx^2 \leq x^3 + xy^2$, then $y \leq x$. **[25 marks]**

(iv) By using the method of proof by cases, show that if n is an even integer, then $n = 4k$ or $n = 4k - 2$ for some integer k. **[20 marks]**

6. (i) Let D_n be a set of divisors of n, where $n \in \mathbb{N}$. Define operations $+$, $\bullet$ and $'$ as $a + b = lcm\,\{a, b\}$, $a \bullet b = \gcd\{a, b\}$, $a' = \frac{n}{a}$. Prove that D_8 is not a Boolean algebra. **[35 marks]**

(ii) (a) Find the output of the following combinatorial circuit in terms of inputs x, y and z.



(b) Draw a combinatorial circuit by simplifying the Boolean expression obtained in part (a). **[35 marks]**

(iii) Find the sum-of-products expansion for the Boolean function $F(x, y, z) = \bar{x} + xy + yz$. **[30 marks]**

-------------End of the question paper-------------