UNIVERSITY OF KELANIYA – SRI LANKA
Centre for Distance and Continuing Education

Bachelor of Science (General) External Degree Examination - 2024
April 2026
Faculty of Science

APPLIED MATHEMATICS
AMAT 17543– Numerical Methods I

No. of Questions: Six (06) **No. of Pages: Five (05)** **Time: Two & half $\left(2\frac{1}{2}\right)$ hrs**

Answer **Five (05)** Questions only.

Programmable calculators are not allowed

1. (a) Explain briefly what is meant by the following terms:

(i) significant digits [05 marks] (ii) relative error [10 marks]

(iii) absolute error [10 marks] (iv) truncation error [10 marks]

(b) (i) What is meant by 'floating point numbers? [10 marks]

(ii) Assume that β is the base, t is the mantissa length, $[L, U]$ is the exponent range of a certain floating-point system.

(α) How many normalized floating-point numbers are there in this system? [10 marks]

(β) What is the smallest positive normalized number (UFL)? [05 marks]

(γ) What is the largest positive normalized number (OFL)? [05 marks]

(iii) Let $\beta = 2, t = 3, U = 1,$ and $L = -1$ for a certain floating-point system.

Write down all the numbers of this system in a base ten form. [25 marks]

Write down UFL and OFL. [10 marks]

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2. (a) Explain briefly the Bisection method for finding approximate solutions to non-linear equation of the type $f(x) = 0$ in $[a, b]$. [25 marks]

(b) Write down two advantages and two disadvantages of Bisection method. [20 marks]

(c) Let α be the root of the given equation $f(x) = 0$ and let x_{n-1}, x_n, x_{n+1} are three consecutive values generated by the Bisection method. Show that the order of convergence

$$\rho = \frac{\log\left|\frac{x_{n+1} - \alpha}{x_n - \alpha}\right|}{\log\left|\frac{x_n - \alpha}{x_{n-1} - \alpha}\right|}.$$ [15 marks]

(d) Consider the function defined by

$$f(x) = x^3 - 5x^2 - 34\,x + 80\,.$$

(i) Show that $f(x)$ has a root in $[1,3]$. [10 marks]

(ii) Determine the bound for the number of iterations to solve the above problem with the accuracy $\varepsilon = 10^{-5}$. [20 marks]

(iii) Apply the Bisection method to find a root of $f(x)$ in $[1,3]$ performing four steps. [10 marks]

3. (a) Using a graphical method, derive the Newton-Raphson formula for finding an approximate solution to a non-linear equation of the type $f(x) = 0$. [25 marks]

(b) Show that the Newton-Raphson process has a quadratic convergence. [30 marks]

(c) Use the Newton-Raphson method to find the root of $x^2 - 1 - \sin x = 0$. Take initial value -0.6 and carryout four iterations.
(You are expected to maintain accuracy to five decimal places in your every answer.) [45 marks]

4. (a) By applying Newton's backward difference formula

$$P_k = y_0 + \frac{k}{1!}\nabla y_0 + \frac{k(k+1)}{2!}\nabla^2 y_0 + \frac{k(k+1)(k+2)}{3!}\nabla^3 y_0 + \cdots$$

obtain the expressions for the first and the second derivatives of P_k. [40 marks]

(b) The function $y = \ln(\sin\theta)$ takes the values given by the following table.

θ^o	1	6	11	16	21	26	31
y	-4.048277	-2.258295	-1.656482	-1.288669	-1.026195	-0.824689	-0.663514

Using the results in part (a) find the values of $\cot\theta$ and $-\text{cosec}^2\theta$ when $\theta = 29^o$. [60 marks]

(You are expected to maintain accuracy to six decimal places in your every answer.)

5. (a) The values of the function $y = f(x)$ at the points $x_0, x_1, \dots, x_n$ are known. The Lagrange's Interpolation polynomial is given by

$$P(x) = \sum_{i=0}^{n} L_i(x) y_i \text{ where}$$

$$L_i(x) = \frac{(x - x_0)(x - x_1)\dots(x - x_{i-1})(x - x_{i+1})\dots(x - x_n)}{(x_i - x_0)(x_i - x_1)\dots(x_i - x_{i-1})(x_i - x_{i+1})\dots(x_i - x_n)}.$$

Show that, if $(n+1)$th derivative of $f(x)$ exists in (x_0, x_n), the error in using Lagrange's Interpolation polynomial instead of $y = f(x)$ is

$$\mathcal{E}(x) = \frac{\Pi(x) f^{(n+1)}(c)}{(n+1)!}$$

for some $c \in (x_0, x_n)$, where $\Pi(x) = (x - x_0)(x - x_1)\dots(x - x_n)$.

[35 marks]

(b) (i) Determine the Lagrange Interpolation polynomial $P(x)$ that Interpolates $y(x) = x\sin x - 3\sin x$ at the nodes $x_i \in \{0, 1, 2, 3\}$.

[20 marks]

(ii) Estimate $y\left(\frac{3}{2}\right)$ using $P(x)$. [05 marks]

(iii) Use $\max_{[0,3]} |y^{(4)}(x)|$ to determine an upper bound for the error in $P\left(\frac{3}{2}\right)$.

Given that $y^{(4)} = y - 4\cos x$ and $x + 5\tan x = 3 \Rightarrow x \approx 0.46865$.

[35 marks]

(iv) Compare the upper bound in (iii) with the actual error.

(You are expected to maintain accuracy to four decimal places in your every answer.) [05 marks]

6. (a) Write down the normal equations for the least square polynomial of degree one. [10 marks]

(b) Andrade's equation has been proposed as a model of the effect of temperature on viscosity following:

$\mu = De^{\frac{C}{T}}$; where μ represents the viscosity, T is the temperature, and D & C are coefficients. The following information has been obtained experimentally:

$T(K)$	90	80	70	60	50
$\mu\ (Ns/m^2)$	0.001787	0.001307	0.001002	0.0007975	0.0006529

(i) In a **maximum of two sentences**, briefly describe why non-linear data such as that shown in the above set needs to be linearized to solve for the non-linear coefficients. [10 marks]

(ii) Linearize the non-linear model proposed above. Clearly state the transformations you use and ensure you show ALL steps and working. [10 marks]

(iii) Relate the non-linear variables to the linear variables as follows: [20 marks]

$$y \quad = \quad a_0 \quad + \quad a_1 \quad x.$$

Linearized model: ☐ = ☐ + ☐ ☐

(iv) Assume you obtained the values in the table below. Calculate the linear coefficients a_0 and a_1. Show your working. [10 marks]

i	x_i	y_i	$x_i y_i$	x_i^2
SUM	0.075	-34.35	-0.518	0.0012
MEAN	0.015	-6.87	-0.1035	-0.1035

(v) From your results in part (iv), calculate the non-linear coefficients.[10 marks]

(vi) Write down the non-linear model. [05 marks]

Continued...

(vii) Consider the following relationship between the non-linear variables and the linear variables: [10 marks]

	y	=	a_0	+	a_1	x.
Linearized model:	$1/\mu$	=	0	+	1	$(1/D)e^{\frac{-C}{T}}$

Is it possible to determine the non-linear coefficients using the above transformations? If yes, provide the necessary steps (maximum 3 steps). If no, provide the reason(s) for why it is not possible.

(viii) A colleague proposes that a better non-linear model to describe the viscosity (μ) to temperature (T) by: $\mu = \alpha T^5 e^{\beta T^3}$.
Linearize the non-linear equation above and relate the non-linear variables to the linear variables, and the non-linear coefficients to the linear coefficients. Show ALL your working. [10 marks]

Relate the non-linear variables to the linear variables as follows: [05 marks]

	y	=	a_0	+	a_1	x.
Linearized model:		=		+		

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