University of Kelaniya - Sri Lanka
Center for Distance & Continuing Education
Bachelor of Science(General) External
First year second semester examination - 2024 (2026 March)
(New Syllabus)
Faculty of Science

Applied Mathematics
AMAT 17532 - Vector Methods in Geometry

No.of Questions: Five(05) No.of Pages: Three(03) Time: Two(2)hrs

Answer Four(04) Questions Only

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1. The points A and B have position vectors $\mathbf{a} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$, $\mathbf{b} = -\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$ respectively.

   (a) Determine a vector equation of the line l_1, passing through A and B in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{d}$, where λ is a scalar parameter.

   (b) The line l_2 has vector equation

   $$\mathbf{r} = \mathbf{i} + 2\mathbf{j} - \mathbf{k} + \mu(\mathbf{i} + \mathbf{j} - \mathbf{k}),$$

   where μ is a scalar parameter.

   i. Show that l_1 and l_2 intersect.

   ii. Find the coordinates of the point of intersection W.

   (c) Find the length of the line segment AW.

   (d) Compute the angle θ which is the acute angle between l_1 and l_2.

2. (a) i. The plane P_1 contains the points $(2, 1, 3)$, $(0, 4, 1)$, $(1, -1, 2)$. Find its equation in Cartesian form.

   ii. The plane P_2 contains the point $(1, 2, -1)$ and has normal vector $\mathbf{i} - \mathbf{j} + 2\mathbf{k}$. Find its equation in Cartesian form.

   iii. The point $(p, 1, q)$ lies in both the planes P_1 and P_2. Find the values of p and q, and express the equation of the line of intersection of the two planes going through this point in the form

   $$\mathbf{r} = \mathbf{a} + \lambda\mathbf{b},$$

   where λ is a parameter.

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(b) i. Suppose a plane passes through a point $a = (x_0, y_0, z_0)$ and is normal to the non-zero vector $\mathbf{n} = A\mathbf{i} + B\mathbf{j} + C\mathbf{k}$, then show that the equation of the plane is given by

$$Ax + By + Cz = D,$$

where D is any constant.

ii. Find the vector equation of the plane that passes through the point $P(3, 2, 2)$ and has normal vector $\mathbf{n} = 2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$.

3. (a) Find the equation of the sphere whose center is at the point $(1, 2, -1)$ and whose radius is 3.

(b) A space curve is defined by

$$x = 2\cos t, \qquad y = 2\sin t, \qquad z = 3t.$$

Find

i. the unit tangent vector $\mathbf{T}$,

ii. the principal normal vector $\mathbf{N}$ and the curvature κ,

iii. the binormal vector $\mathbf{B}$ and the torsion τ,

iv. the equations of the osculating plane, the normal plane, and the rectifying plane at the point corresponding to $t = \frac{\pi}{4}$.

4. (a) State and prove Frenet-Serret formulas.

(b) show that

i. $\ddot{\mathbf{r}} = \ddot{s}\,\mathbf{T} + \kappa\dot{s}^2\,\mathbf{N}$

ii. $\dddot{\mathbf{r}} = (\dddot{s} - \kappa^2\dot{s}^3)\mathbf{T} + (\dot{\kappa}\dot{s}^2 + 3\kappa\dot{s}\ddot{s})\mathbf{N} + \kappa\tau\dot{s}^3\mathbf{B}$

iii. Show that the curvature κ and torsion τ for a curve $\mathbf{r} = \mathbf{r}(t)$ can be written in the form

$$\kappa = \frac{||\dot{\mathbf{r}} \times \ddot{\mathbf{r}}||}{||\dot{\mathbf{r}}||^3}$$

and

$$\tau = \frac{[\dot{\mathbf{r}}\ \ddot{\mathbf{r}}\ \dddot{\mathbf{r}}]}{||\dot{\mathbf{r}} \times \ddot{\mathbf{r}}||^2}$$

where $(\dot{\ })$ denotes $\frac{d}{dt}$ and t is any parameter other than the arc length s.

(c) Consider the helix

$$r(t) = (a\cos t, a\sin t, bt),$$

where a and b are constants and t is a parameter.
Using part (b), find

i. the curvature κ

ii. the torsion τ.

Continued...

5. (a) In the usual notation, a point in space in cylindrical coordinates can be described by

$$p = (r\cos\theta,\ r\sin\theta,\ z).$$

i. Obtain the scale factors h_r, h_θ, and h_z.

ii. Find the unit vectors $\mathbf{e}_r, \mathbf{e}_\theta$, and $\mathbf{e}_z$.

iii. Hence show that the coordinate system is orthogonal.

(b) Let $\mathbf{F}(u, v, w)$ and $\mathbf{G}(u, v, w)$ be vector fields defined by

$$\mathbf{F}(u, v, w) = u\cos v\ \mathbf{e}_u + u\sin v\ \mathbf{e}_v + u\sin w\ \mathbf{e}_w$$

$$\mathbf{G}(u, v, w) = v\ \mathbf{e}_u + u^2\ \mathbf{e}_v + w^2\ \mathbf{e}_w.$$

Find,

i. $\nabla \times \mathbf{F}$

ii. $\nabla \cdot \mathbf{G}$

— End of Examination —