University of Kelaniya - Sri Lanka
Center for Distance & Continuing Education
Bachelor of Science (General) External
First year first semester examination - 2024 (September 2025)
(New Syllabus)
Faculty of Science

Applied Mathematics
AMAT 16513 - Vector Analysis

No.of Questions: Six(06) **No.of Pages: Three(03)** **Time: Two & half($2\frac{1}{2}$)hrs**

Answer **Five(05)** Questions Only.

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1. (a) Find the equation of the sphere that passes through the point $(3, 2, 4)$ and has center $(7, 8, 2)$.

   (b) Find the vector $\vec{b}$ whose scalar projection onto $\vec{a} =< 3, 0, 2 >$ is 3.

   (c) Given four points $P = (3, 0, 1)$, $Q = (-1, 2, 5)$, $R = (5, 1, -1)$, and $S = (0, 4, 2)$, compute the volume of the parallelepiped formed by the adjacent edges PQ, PR, and PS.

   (d) Find the equation of the plane that contains the line $x = 1 + t$, $y = 2 - t$, $z = 4 - 3t$ and is parallel to the plane $5x + 2y + z = 1$.

   (e) Let two non-zero vectors $\vec{v_1}$ and $\vec{v_2}$ have magnitudes w and z, respectively. Given that $x = |\vec{v_1} \times \vec{v_2}|$ and $y = \vec{v_1} \cdot \vec{v_2}$. Show that $x^2 + y^2 = w^2 z^2$.

2. (a) Sketch the surface $y = 2x^2 + z^2$ by using traces and intercepts.

   (b) Suppose $\vec{A}$, $\vec{B}$ are vectors. Prove that

   $$\vec{A} \times \frac{d^2\vec{B}}{dt^2} - \frac{d^2\vec{A}}{dt^2} \times \vec{B} = \frac{d}{dt}\Big(\vec{A} \times \frac{d\vec{B}}{dt} - \frac{d\vec{A}}{dt} \times \vec{B}\Big).$$

   (c) A particle moves in space under the influence of an acceleration given by $\vec{a} =< 6t, \cos t, e^t >$. If the initial velocity of the particle at $t = 0$ is $\vec{v}(0) =< 2, 1, 0 >$, find the velocity $\vec{v}(t)$ at any time t.

   (d) In the usual notation, derive the formula for tangent and normal components of the acceleration $a_T = \dfrac{\vec{r'}(t) \cdot \vec{r''}(t)}{|\vec{r'}(t)|}$ and $a_N = \dfrac{|\vec{r'}(t) \times \vec{r''}(t)|}{|\vec{r'}(t)|}$ respectively.

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3. (a) The position vector of moving particle is $\vec{r}(t) = < t, 4\cos t, 4\sin t >$.

   (i) Find a unit tangent vector $\vec{T}$, the unit normal vector $\vec{N}$ and the binormal vector $\vec{B}$ of the particle, in the direction of motion.

   (ii) Find the curvature.

   (iii) Obtain the equation of the osculating plane and normal plane at $t = \pi$.

   (b) Show that $\frac{d\vec{N}}{ds} + \kappa\vec{T}$ is perpendicular to both $\vec{T}$ and $\vec{N}$, where $\vec{T}$ is the unit tangent vector and $\vec{N}$ is the unit normal vector.

   Hint: Consider the Frenet- Serret Formula $\frac{d\vec{N}}{ds} = -\kappa\vec{T} + \tau\vec{B}$, where $\vec{B}$ is the binormal vector. Also κ and τ are curvature and torsion respectively and consider them as constants.

4. (a) Evaluate $\int_C xy^3\, ds$, where C is the arc of the circle $x^2 + y^2 = 9$ lying in the first quadrant.

   (b) Given the vector field $\vec{F} = < 2xy, x^2 + 3y^2 >$.

   (i) Show that $\vec{F}$ is a conservative vector field.

   (ii) Is vector field $\vec{F}$ irrotational?

   (iii) Find a potential function $f(x, y)$ such that $\vec{F} = \nabla f$.

   (iv) Evaluate $\int_C \vec{F} \cdot d\vec{r}$, where C is the curve $\vec{r}(t) = (7t + t^{2025})\ \hat{i} + \frac{t}{\sqrt{2-t}}\ \hat{j}$ with $0 \leq t \leq 1$.

   (c) Evaluate $\oint \vec{F} \cdot d\vec{r}$, where $\vec{F} = < \sin^{-1}(x^3 + 1) + \ln(x + 3), \tan y + \frac{1}{2}x^2 >$ and C is the boundary of the region enclosed by the curves $y = x^2$, $y = 3$ and $x = 0$ with counterclockwise orientation.

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5. (a) Find the gradient vector field of the function $f(x, y, z) = x\cos(yz)$.

(b) Evaluate the surface integral $\iint_S z\, dS$ where S is the upper hemisphere $x^2 + y^2 + z^2 = 16$, $z \geq 0$ and has upward orientation.

(c) (i) State the Stokes' theorem.

(ii) Use Stokes' theorem to evaluate $\int_C \vec{F} \cdot d\vec{r}$, where $\vec{F} = z\hat{\imath} + x\hat{\jmath} + y\hat{k}$ and S is the part of the plane $x+y+z = 1$, that lies in the first octant $(x, y, z \geq 0)$, oriented upward. Let C be the boundary curve of the S, with the vertices $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$ oriented counterclockwise as viewed from above.

6. (a) Prove

(i) $\nabla\times(\nabla\phi) = 0$ (Curl grad $\phi = 0$), (ii) $\nabla\cdot(\nabla\times\vec{F}) = 0$ (div Curl $\vec{F} = 0$),

where $\vec{F} = F_1\,\hat{\imath} + F_2\,\hat{\jmath} + F_3\hat{k}$ is a vector, ϕ is a scalar function and assume both ϕ and $\vec{F}$ have continuous second partial derivatives.

(b) (i) State the Divergence theorem.

(ii) Use Divergence theorem to calculate the surface integral of the vector field $\vec{F} = xy^2\,\hat{\imath} + yz^2\,\hat{\jmath} + zx^2\,\hat{k}$, where S is the surface of the solid bounded by the cylinder $x^2 + y^2 = R^2$ and the planes $z = 0$ and $z = h$. Let R and h be positive constants.

— End of Examination —