UNIVERSITY OF KELANIYA SRI LANKA
Center for Distance and Continuing Education

Bachelor of Science (General) External
First Year First Semester Examination 2024
2025 – September
(New Syllabus)
Faculty of Science

APPLIED MATHEMATICS

AMAT 16522/R- Mechanics I

No. of Questions: Six (06) No. of Pages: Three (03) Time: Two (02) hours

Answer Four (04) Questions only.

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1. (a) Define an inertial frame in Newtonian Mechanics.

   (b) Consider two reference frames S and S' which were coincide at $t = 0$. Suppose that the frame S' is moving with the velocity v in the direction of the positive $x-$axis relative to the frame S. In the usual notation, obtain the Lorentz transformation equations between the two reference frames S and S' in the form

$$x' = \gamma\,(x - vt)\,,$$
$$y' = y,$$
$$z' = z,$$
$$t' = \gamma\left(t - \frac{vx}{c^2}\right) \quad \text{where } \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}.$$

   Hence, show that the time dilation formula takes the form

$$\tau = \tau'\sqrt{1 - \frac{v^2}{c^2}},$$

   where τ' is the time interval of which an event take place in the frame S' and τ is the the time interval that the event take place in the frame S.

   (c) A space ship S_1 is on its way to approach X-centauri when it passes space ship S_2 at relative speed v. The captain in S_1 sends a radio signal that exists only $2s$ according to its clock. According to the clock in S_2, the signal exists only $1.89s$. Determine the velocity v.

2. (a) In the usual notation, using polar coordinates, obtain the radial and transverse components of acceleration $\left(\ddot{r} - r\dot{\theta}^2\right)$ and $\frac{1}{r}\frac{d(r^2\dot{\theta})}{dt}$ of a particle moving in a plane, respectively.

(b) A particle moves along a circle $r = 2a\cos\theta$ in such a way that its acceleration towards the origin is always the zero. Show that the transverse component of the acceleration varies by an amount of $cosec^5\theta$.

(c) The position vector of a particle is given by $\underline{r}_1 = 2t\underline{i} - 3e^t\underline{j} + 3t^3\underline{k}$, where t is the time variable. Determine the power P developed by the force $\underline{F} = \underline{i} + 4\underline{j} + 4\underline{k}$ which acts on the particle when $t = 3$.

3. (a) In the usual notation, obtain the formulae

$$\frac{d^2u}{d\theta^2} + u = \frac{F}{h^2u^2}, \text{ and } \dot{\theta} = hu^2,$$

for a motion of a particle that describes an orbit, under the central force F per unit mass, where $u = \frac{1}{r}$ and h is a constant.

(b) A particle describes the orbit $r = \frac{a}{(1+\cos n\theta)}$ under the central force F to the pole, where a and n are constants. Show that

$$F = \frac{h^2}{r^2}\left(\frac{1}{r}(1-n^2) + \frac{n^2}{a}\right).$$

(c) Prove that the motion of a particle of mass m under a central force towards a fixed point is always a plane curve.

4. (a) Consider the motion described by the **question 3(a) above**. Show that

i. $\dot{r} = -h\frac{du}{d\theta}$,

ii. $v^2 = \dot{r}^2 + (r\dot{\theta})^2 = h^2\left[u^2 + \left(\frac{du}{d\theta}\right)^2\right]$, where v is the velocity of the particle.

(b) Suppose that a particle of unit mass describes an orbit $r = \alpha e^{\beta\theta}$, where α and β are some constants. It is given that initially ($t = 0$), $r = r_0$. In subsequent motion, find

i. the law of force F,

ii. the velocity of particle v,

iii. a formula for total energy (E) assuming that F is conservative.

(c) In the usual notation, the pedal equation for the motion under the force F is given by $\frac{h^2}{p^3}\frac{dp}{dr} = F$. Derive an equation for p for the orbit described in above part (b) (.i.e. for $r = \alpha e^{\beta\theta}$).

5. (a) Under the influence of a force field, a particle of mass m moves along the ellipse which is given by $\underline{r} = a\cos\omega t\underline{i} + b\sin\omega t\underline{j}$, where a, b, ω are constants. If $\underline{p}$ is the momentum of the particle, show that

$$\underline{r} \times \underline{p} = m\,a\,b\,\omega\,\underline{k}.$$

(b) A particle describes an ellipse of eccentricity e about a center of force at a focus. Prove in the usual notation

$$v^2 = \frac{h^2}{l}\left(\frac{2}{r} - \frac{(1-e^2)}{l}\right).$$

Hence, determine the new velocity if the particle changes its orbit from ellipse to parabola.

6. (a) Assume in the usual notation $\dfrac{d\underline{A}}{dt} = \dfrac{\partial \underline{A}}{\partial t} + \underline{\omega} \times \underline{A}$ for a vector $\underline{A}$ with respect to two references of frames, one is rotating relative to the other (here $\dfrac{\partial \underline{A}}{\partial t}$ is the derivative of $\underline{A}$ with respect to the rotating frame). Then prove that

$$\frac{d^2\underline{r}}{dt^2} = \frac{\partial^2 \underline{r}}{\partial t^2} + \frac{\partial \underline{\omega}}{\partial t} \times \underline{r} + 2\underline{\omega} \times \frac{\partial \underline{r}}{\partial t} + \underline{\omega} \times (\underline{\omega} \times \underline{r})$$

where $\underline{r}$ is the position vector of the particle with respect to the origin on the surface of the earth and $\underline{\omega}$ is the angular velocity of the earth's diurnal rotation.

(b) An xyz coordinate system is rotating with respect to an XYZ coordinate system with the same origin and is assumed to be fixed in space. The angular velocity of the xyz system relative to the fixed system is given by $\underline{\omega} = t\underline{i} + 2t\underline{j} + 3\underline{k}$. The position vector of a particle at time t as observed in xyz system is given by $\underline{r} = \underline{i} + 2t\underline{j} + 3\underline{k}$. Find the acceleration of the particle as seen by an observer in the fixed system.

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