University of Kelaniya – Sri Lanka
Centre for Distance and Continuing Education

Bachelor of Science (General) External Degree Examination - 2023
June 2025
Faculty of Science

Applied Mathematics
AMAT 17543– Numerical Methods I

No. of Questions: Six (06) **No. of Pages: Three (03)** **Time: Two & half $\left(2\frac{1}{2}\right)$ hrs**

Answer **Five (05)** Questions only.

Programmable calculators are not allowed.

1. (a) What is meant by stability in numerical analysis? **[05 marks]**

   (b) Let E_a and E_b are the absolute errors of two numbers a and b respectively. Assuming that $\frac{E_b}{b} \ll 1$, show that the absolute error in $a/_b$ is approximately given by $\frac{a}{b}\left(\frac{E_a}{a} - \frac{E_b}{b}\right)$. **[20 marks]**

   (c) Let $f(x) = 2x\cos(2x) - (x-2)^2$.

      (i) Find the third order Taylor polynomial $P_3(x)$ about $x_0 = 0$, and use it to approximate $f(0.4)$. **[50 marks]**

      (ii) Use the error formula in Taylor's Theorem to find an upper bound for the error $|f(0.4) - P_3(0.4)|$. **[25 marks]**

2. (a) (i) Define the term "ill-conditioned problem". **[10 marks]**

      (ii) Give an example of a polynomial that has ill-conditioned zeros.

   [10 marks]

   (b) Show that the system

   $$\begin{pmatrix} 0.96 & -1.23 \\ 4.91 & -6.29 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -0.27 \\ -1.38 \end{pmatrix}$$ is ill-conditioned. **[30 marks]**

   (c) (i) Give two reasons to consider pivoting strategy when solving a system of linear equations using the method of Gaussian elimination.

   [15 marks]

      (ii) Solve the following linear system of equations using the method of Gaussian elimination with pivoting. **[35 marks]**

   $$-2x_2 + 2x_3 = 4, \quad x_1 - 2x_2 + x_3 = 3, \quad -2x_1 + 2x_2 + 4x_3 = 0$$

Continued...

3. (a) (i) Derive the Newton-Raphson formula graphically for finding an approximate solution to non-linear equations of the type $f(x) = 0$.
[20 marks]

(ii) By applying the Newton-Raphson method to $f(x) = x^2 - {}^1/_c$, obtain an iterative formula, where c is any positive number. **[10 marks]**

(iii) Using the iterative formula obtained in (ii) above show that $e_i = \frac{e_{i-1}^2}{2x_{i-1}}$,
where $e_i = x_i - \frac{1}{\sqrt{c}}, i = 0,1,2, \ldots$

Hence show that the iterative formula obtained in (ii) is quadratically convergent. **[30 marks]**

(iv) Estimate the value of $\sqrt{2}$ to four decimal places using the Newton-Raphson method. (Use three iterations). **[20 marks]**

(b) The operator Δ is defined as follows.

$\Delta f(x) = f(x+h) - f(x)$, where h is the interval of differencing.

Show that $\Delta \log f(x) = \log\left[1 + \frac{\Delta f(x)}{f(x)}\right]$. **[20 marks]**

4. (i) Derive the normal equation for the least squares polynomial of degree one.
[50 marks]

(ii) The number of radioactive nuclei in the material at time $t, N(t)$ is given by,

$N(t) = N_0 e^{-\lambda t}$, where N_0 is the initial number of radioactive nuclei and λ is a constant. Use the data given in the following table and the method of least squares to find the values N_0 and λ.

t(time)	1	2	3	4	5	6
N	4887	3757	3923	2742	2655	2001

[50 marks]

5. (a) Suppose that the values of the function f at the points x_0, x_1, x_2, x_3, x_4 are known.

Obtain the 5-point Lagrange's interpolation formula in the form

$P(x) = \sum_{i=0}^{4} L_i(x) f(x_i)$, where each $L_i(x)$ is a quadratic polynomial in x.

[35 marks]

Continued...

(b) (i) Use the data given in the following table to find the 4th order Lagrange's interpolation polynomial. **[35 marks]**

i	**0**	**1**	**2**	**3**	**4**
x_i	1	2	3	4	5
y_i	11.6	16.2	16.8	13.5	7.3

(ii) Also use the Newton's method to find maximum value of y. **[30 marks]**

6. (a) Explain the method of fixed-point iteration for finding a root of $f(x) = 0$.

[25 marks]

(b) (i) Show that $f(x) = \sin(x^2) = 0$ can be expressed as $x = g_1(x)$, where

$g_1(x) = \frac{\sin(x^2)}{x^2} + x.$

Use fixed-point iteration theorem to show that g_1 has a unique fixed-point in $[1.7, 1.8]$. **[25 marks]**

(ii) Apply fixed-point iteration method to find the fixed-point of g_1. Take $x_0 = 2$ and impose a tolerance of 10^{-4}. **[30 marks]**

(iii) Now replace $g_1(x)$ by $g_2(x) = \frac{\sin(x^2)}{x} + x.$

Is it possible to have a fixed-point to g_2? Explain your answer.

[20 marks]

----- **End of question paper** -----