University of Kelaniya - Sri Lanka
Center for Distance & Continuing Education
Bachelor of Science(General) External
First year second semester examination - 2023 (2025 March)
(New Syllabus)
Faculty of Science

Applied Mathematics
AMAT 17532 - Vector Methods in Geometry

**No.of Questions: Five(05)** **No.of Pages: Three(03)** **Time: Two(2)hrs**

**Answer Four(04) Questions Only**

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1. (a) Consider the two lines $\mathbf{r_1} = \mathbf{a}+\lambda\mathbf{m}$ and $\mathbf{r_2} = \mathbf{b}+\mu\mathbf{n}$ where $\mathbf{a} = 5\mathbf{i}+\mathbf{j}+2\mathbf{k}$, $\mathbf{b} = -\mathbf{i}+7\mathbf{j}+8\mathbf{k}$, $\mathbf{m} = -4\mathbf{i}+1\mathbf{j}-1\mathbf{k}$ and $\mathbf{n} = 2\mathbf{i}-5\mathbf{j}-7\mathbf{k}$.

   i. Show that the two lines intersect.

   ii. Find the position vector of the point $P$ of their intersection.

   iii. If the position vector of a point $Q$ is $3\mathbf{i}+7\mathbf{j}-2\mathbf{k}$, show that $PQ$ is perpendicular to $AB$ where $A$ and $B$ are the points with the position vectors $\mathbf{a}$ and $\mathbf{b}$.

   (b) i. Find the vector equation of a line which passes through the point $A$, whose position vector is $2\mathbf{i}+4\mathbf{j}+7\mathbf{k}$ and is parallel to the vector $-2\mathbf{i}+3\mathbf{j}+\mathbf{k}$.

   ii. What are the point coordinates where this line intersects the $xy$ plane?

2. (a) i. Find an equation of the plane normal to the vector $2\mathbf{i}-\mathbf{j}$ that contains the line

$$l : 2\mathbf{i}+\mathbf{j}+3\mathbf{k}+t(\mathbf{i}+2\mathbf{j}-\mathbf{k})$$

   where $t$ is a parameter.

   ii. Find the equation of the plane containing the same line $l$ and the point with position vector $\mathbf{k}$.

   iii. Find the angle between these two planes.

   (b) Let $\mathbf{a}, \mathbf{b}, \mathbf{c}$ be non-co-planar vectors.

   i. Write down the vector equation of the plane which contains the points $2\mathbf{a}+2\mathbf{b}$, $3\mathbf{a}+2\mathbf{b}+\mathbf{c}$, and $\mathbf{a}+3\mathbf{b}+\mathbf{c}$.

   ii. Determine whether the point $-3\mathbf{a}+2\mathbf{b}+\mathbf{c}$ is also on the plane.

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3. (a) Find equation of the line passing through the point $(5, 4, 3)$ and having direction ratios $-3, 4, 2$.

   (b) Find equation of the sphere whose center is at the point $(1, 2, -1)$ and whose radius is 3.

   (c) Consider the following two skew-lines

$$\frac{x+3}{-4} = \frac{y-6}{3} = \frac{x}{2}; \qquad \frac{x+2}{-4} = \frac{y}{1} = \frac{x-7}{1}$$

   i. Write down the vector equations of above lines.

   ii. Find the length of the shortest distance between them.

4. (a) Find the unit tangent vector, the unit normal vector, and the unit binormal vector of the circle of radius $a$ in the $xy-$plane, parameterized as

$$r(\theta) = (a\cos(\theta), a\sin(\theta), 0)$$

   at $\theta = \dfrac{\pi}{4}$.

   (b) State Frenet-Serret formulas.

   (c) Show that the curvature $\kappa$ and torsion $\tau$ for a curve $\mathbf{r} = \mathbf{r}(t)$ can be written in the form

$$\kappa = \frac{||\dot{\mathbf{r}} \times \ddot{\mathbf{r}}||}{||\dot{\mathbf{r}}||^3}$$

   and

$$\tau = \frac{[\dot{\mathbf{r}} \;\; \ddot{\mathbf{r}} \;\; \dddot{\mathbf{r}}]}{||\dot{\mathbf{r}} \times \ddot{\mathbf{r}}||^2}$$

   (d) i. Find the curvature and the torsion of the twisted cubic $\mathbf{r} = \left(2t, t^2, \dfrac{t^3}{3}\right)$ at $t = 1$.

   ii. Derive the equation of the osculating plane.

5. (a) Find the equation of the tangent plane to $z = 3x^2 - xy$ at the point $(1, 2, 1)$.

   (b) The coordinates $(u, v, w)$, are related to Cartesian coordinates $(x, y, z)$ by the following equations.

$$x = 2u + 3v - w$$
$$y = v - 5w$$
$$z = u + 4w$$

   i. Obtain the scale factors $h_u, h_v$, and $h_w$.

   ii. Find the unit vectors $\mathbf{e}_u, \mathbf{e}_v$, and $\mathbf{e}_w$.

   iii. Show that the coordinate system $(u, v, w)$ is orthogonal.

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(c) Let $\mathbf{F}(u, v, w)$ and $\mathbf{G}(u, v, w)$ be

$$\mathbf{F}(u, v, w) = u \sin v \, \mathbf{e}_u + u \cos v \, \mathbf{e}_v + u \cos w \, \mathbf{e}_w$$

$$\mathbf{G}(u, v, w) = v \, \mathbf{e}_u + u \, \mathbf{e}_v + w \, \mathbf{e}_w.$$

Find,

i. $\nabla \times \mathbf{F}$

ii. $\nabla \cdot \mathbf{G}$

— End of Examination —