University of Kelaniya – Sri Lanka
Centre for Distance & Continuing Education
Bachelor of Science (General) External
First year second semester examination - 2019
(New Syllabus)
2023 March
Faculty of Science
APPLIED MATHEMATICS

Vector Methods in Geometry – AMAT E 17532

No. of Questions: Fine (05) **No. of Pages: Three(03)** **Time Allowed: Two(02)hrs**

Answer Four (04) questions only.

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01. (a) Determine the vector equation of the straight line passing through the point with position vector $\underline{i} - 3\underline{j} + \underline{k}$ and parallel to the vector $2\underline{i} - 3\underline{j} - 4\underline{k}$.

Express the vector equation of the straight line in standard cartesian form.

(b) Show that there is a point common to the two straight lines $\underline{r} = 3\underline{j} + 2\underline{k} + s\left(2\underline{i} + 2\underline{j} + 3\underline{k}\right)$ and $\underline{r} = -2\underline{i} + 2\underline{j} + 3\underline{k} + t\left(9\underline{i} - 3\underline{j} - 6\underline{k}\right)$, where s and t are parameters.

Determine the coordinates of the common point.

(c) Find the equation of the plane normal to the vector $\underline{i} + \underline{j}$ containing the line l with the equation $\underline{r} = \underline{i} + \underline{j} + \underline{k} + t\left(\underline{i} - \underline{j} - \underline{k}\right)$, where t is a parameter.

Find also the equation of the plane containing line l and point $\underline{j}$.

Show that the angle between these planes is $\frac{\pi}{3}$.

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02. (a) State and prove the Serret-Frenet formulae.

(b) Consider the space curve $\underline{r} = \left(3\cos\frac{s}{5}, 3\sin\frac{s}{5}, \frac{4s}{5}\right)$.

(i) Find the unit tangent vector $\underline{t}$, unit principal normal vector $\underline{n}$ and the unit binormal vector $\underline{b}$ at any point of the curve.

(ii) Obtain the equations of the rectifying and normal planes to the curve at the point corresponding to $s = 1$.

03. (a) Prove, in the usual notation, that the curvature κ and the torsion τ of a space curve $\underline{r} = \underline{r}(t)$ can be written in the form

$$\kappa = \frac{|\dot{\underline{r}}\times\ddot{\underline{r}}|}{|\dot{\underline{r}}|^3} \quad \text{and} \quad \tau = \frac{\dot{\underline{r}}\times\ddot{\underline{r}}\cdot\dddot{\underline{r}}}{|\dot{\underline{r}}\times\ddot{\underline{r}}|^2},$$

where t is any parameter other than s.

(b) Consider the space curve $\underline{r} = \left(a(3t - t^3), 3at^2, a(3t + t^3)\right)$, where a is a constant.

Show that the curvature κ and the torsion τ of the space curve are given by

$$\kappa = \tau = \frac{1}{3a(1+t^2)^2}.$$

04. (a) Define, in the usual notation, the scale factors h_1, h_2, h_3 and the unit vectors $\underline{e}_1, \underline{e}_2, \underline{e}_3$ for a curvilinear coordinate system u_1, u_2, u_3.

Let ϕ be a scalar function of the curvilinear coordinates u_1, u_2, u_3.

In the usual notation, show that $grad\ \phi$ is given by

$$grad\ \phi = \underline{\nabla}\phi = \frac{1}{h_1}\frac{\partial\phi}{\partial u_1}\underline{e}_1 + \frac{1}{h_2}\frac{\partial\phi}{\partial u_2}\underline{e}_2 + \frac{1}{h_3}\frac{\partial\phi}{\partial u_3}\underline{e}_3.$$

(b) The cartesian coordinates can be written in terms of spherical polar coordinates r, θ and ϕ as

$$x = r\sin\theta\cos\phi,\ \ y = r\sin\theta\sin\phi \text{ and } z = r\cos\theta.$$

(i) Obtain the scalar factors h_r, h_θ and h_ϕ.

(ii) Find the basis vectors $\underline{e}_r, \underline{e}_\theta$ and $\underline{e}_\phi$.

(iii) Calculate $\underline{\nabla}\phi$ and $\underline{\nabla}\left(\frac{1}{r}\right)$.

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(c) In cylindrical coordinates, the position vector of any point can be wriiten in the form

$$\underline{r} = \rho\, cos\phi\, \underline{i} + \rho\, sin\, \phi\, \underline{j} + z\underline{k}.$$

Assume that the base vectors are given by
$\underline{e}_\rho = cos\phi\, \underline{i} + sin\, \phi\, \underline{j}$, $\underline{e}_\phi = -sin\phi\, \underline{i} + cos\, \phi\, \underline{j}$ and $\underline{e}_z = \underline{k}$.

Express the vector $\underline{A} = 2y\underline{i} - z\underline{j} + 3x\underline{k}$ in terms of cylindrical coodinates.

05. (a) A plane curve has been described by the parametric equations

$x(t) = \sqrt{2t+4}$ and $y(t) = 2t + 1$ for $-2 \leq t \leq 6$.

Eliminate the parameter t and sketch the resulting graph.

(b) Skech the graph of polar equation $r = 1 - 2\, cos\theta$. Determine the type of the polar equation.

(c) Write down the implicit form of the surface given by

$$\underline{r}(u, v) = 7 \cos u \sin v\, \underline{i} + 7 \sin u \sin v\, \underline{j} + 7 \cos v\, \underline{k}.$$

Hence identify the suface .

(d) Let the parametric form of a surface $\underline{r}(u, v)$ be

$$\underline{r}(u, v) = (u^2 - v^2)\underline{i} + (u + v)\underline{j} + uv\, \underline{k}.$$

Find the equation of the tangent plane to the surface at the point (1,2).

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